SOLUTION SPECIAL ALGEBRA

1. (c) $2x+\frac{2}{x}=3 \quad x+\frac{1}{x}=\frac{3}{2}$

$$x^3+\frac{1}{x^3}+2=\left(\frac{3}{2}\right)^3-3\times\frac{3}{2}+2=\frac{7}{8}$$

2. (d) $x+\frac{1}{x}=p$

$$x^3+\frac{1}{x^3}=p^3-3p$$

$$\therefore \quad x^6+\frac{1}{x^6}=(p^3-3p)^2-2$$

$$=p^6-6p^4+9p^2-2$$

3. (a) $3x^2+5x+3=0$

$$3x(x+\frac{1}{x})=-5x$$

$$x+\frac{1}{x}=-\frac{5}{3}$$

$$\therefore \quad x+\frac{1}{x}=\left(-\frac{5}{3}\right)^3-3\left(-\frac{5}{3}\right)=\frac{10}{27}$$

4. $a=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}, b=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$a=5-2\sqrt{6}, \qquad b=5+2\sqrt{6}$

$ab=1$ *i.e.* reciprocal, $a+\frac{1}{a}=10$

$$\frac{a^2}{b}+\frac{b^2}{a}=a^3+\frac{1}{a^3}=10^3-3\times10=970$$

5. (a) $\frac{2p}{p^2-2p+1}=\frac{1}{4}$

$$p^2-2p+1=8p$$

$$\text{p}\left(p+\frac{1}{p}\right)=10p$$

$$\therefore \quad p+\frac{1}{p}=10$$

6. (b) $x^2+\frac{1}{x^2}=66 \quad x-\frac{1}{x}=\sqrt{66-2}=\pm8$

$$\text{then } \frac{x^2-1+2x}{x}=\frac{x\left(x-\frac{1}{x}\right)+2x}{x}$$

$$=\frac{8x+2x}{x}=10$$

$$\text{or} =\frac{-8x+2x}{x}=-6$$

$\therefore$ Required value $=(10,-6)$

7. (b) n=7 $4\sqrt{3} \quad \frac{1}{n}=7-4\sqrt{3} \quad n+\frac{1}{n}=14$

$$\left(\sqrt{n}+\frac{1}{\sqrt{n}}\right)^2=n+\frac{1}{n}+2=14+2=16$$

$$\therefore \quad \sqrt{n}+\frac{1}{\sqrt{n}}=\sqrt{16}=4$$

8. (a) $x+\frac{1}{x}=\frac{25}{12}=x-\frac{1}{x}=\sqrt{\left(\frac{25}{12}\right)^2-4}=\frac{7}{12}$

$$x^2+\frac{1}{x^2}=\left(\frac{25}{12}\right)^2-2=\frac{337}{144}$$

$$\text{Then } x^4-\frac{1}{x^4}=\left(x^2+\frac{1}{x^2}\right)\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$$

$$=\frac{337}{144}\times\frac{25}{15}\times\frac{7}{12}=\frac{58975}{20736}$$

9. (a) $x^4+\frac{1}{x^4}=194=x^2+\frac{1}{x^2}=\sqrt{194+2}=14$

$$x+\frac{1}{x}\setminus\sqrt{14+2}=4$$

$$\text{Then } x^3+\frac{1}{x^3}=4^3-3\times4=52$$

10. $\boldsymbol{xy+yz+zx=0}$

put $x=\frac{1}{2}$, $y=-1$ & $z=-1$

$$\text{then } \frac{1}{x^2-yz}+\frac{1}{y^2-zx}+\frac{1}{z^2-xy}=0$$

11. **(c)** $\boldsymbol{\frac{a}{b}+\frac{b}{a}=2}$

$\frac{a}{b}=\frac{b}{a}=1$

i. e., $\qquad a=b$

$\therefore \qquad a-b=0$

12. **(a)**

13. $\boldsymbol{\frac{1}{x^{\frac{1}{4}}}+\frac{1}{\frac{1}{x^4}}=1}$

$$x^{\frac{1}{2}}+\frac{1}{x^{\frac{1}{2}}}=1^2-2=-1$$

$$x+\frac{1}{x}=(-1)^2-2=-1$$

$x^3 = 1$

then $x^{252} + \frac{1}{x^{252}} = (x^3)^{84} \frac{1}{(x^3)^{94}}$

$= 1 + 1 = 2$

14. $\boldsymbol{x + \frac{1}{x} = -\sqrt{3}}$

$x^3 + \frac{1}{x^3} = 0$

$x^6 = -1$

$x^{67} + x^{53} + x^{43} + x^{29} + x^{24} + x^{12} + x^6 + 3$

$= (x^6)^{11} x + \frac{(x^6)^9}{x} + (x^6)^7 . x + \frac{(x^6)^5}{x} + (x^6)^4 + (x^6)^2 + x^6 + 3$

$= -x - \frac{1}{x} - x - \frac{1}{x} + 1 + 1 - 1 + 3$

$= -2x - \frac{2}{x} + 4 = -2\left(x + \frac{1}{x}\right) + 4$

$= -2 \times -(\sqrt{3}) + 4 = 2(\sqrt{3} + 2)$

15. $\boldsymbol{(x+3)^2 + (y-5)^2 + (z+2)^2 = 0}$

$x + 3 = 0$
$y - 5 = 0$ $\Big\}$ $x = -3,\ y = 5,\ z = -2$
$z + 2 = 0$

then $\sqrt{x + y + z} = 0$

16. $\boldsymbol{a = x + y,\quad b = x - y,\quad c = x + 2y}$

$a^2 + b^2 + c^2 - ab - bc - ca$

$= \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2]$

$3(a^2 + b^2 + c^2) = (a + b + c)^2$

$2(a^2 + b^2 + c^2) - 2(ab + bc + ca) = 0$

$a^2 + b^2 + c^2 - ab - bc - ca = 0$

17.(a)

18. $\boldsymbol{x + y = z}$

$x + y + (-z) = 0$

then $x^3 + y^3 + (-z)^3 - 3xy(-z) = 0$

19. (b)	**20.** (c)	**21.** (c)	**22.** (d)	**23.** (b)
24. (c)	**25.** (c)	**26.** (c)	**27.** (b)	**28.** (b)
29. (b)	**30.** (a)	**31.** (d)	**32.** (c)	33. (d)
34. (c)	**35.** (b)	**36.** (b)	**37.** (b)	38. (c