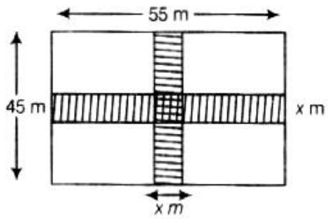
Solutions Mensuration

1. ***(c)*** Given, length = 55m

Breadth = 45m



Area of lawn = $1911 m^2$

Area of rectangular plot

$= 45 \times 55 = 2475\ m^2$

Area of crossroads

$= 2475 - 1911 = 564\ m^2$

Let the width of each crossroad = x m

$\therefore \quad 45x + 55x - x^2 = 564$

$x^2 - 100x + 564 = 0$

$x^2 - 94x + 6x + 564 = 0$

$x(x - 94) - 6(x - 94) = 0$

$(x - 6)(x - 94) = 0$

$x = 6, 94$

[$\because$ 94 is discraded]

$\therefore \quad x = 6$ m

$\therefore$ Width of each of the crossroads = 6 m

2. ***(a)*** Let the side of square be x m.

So, length of rectangle is $(x + 8)$ m

and breadth of rectangle is $(x - 6)$ m.

According to the question,

Area of square = Area of rectangle.

$\Rightarrow \quad x \times x = (x + 8)(x - 6)$

$\Rightarrow \quad x^2 = x^2 + 8x - 6x - 48$

$\Rightarrow \quad x^2 = x^2 + 2x - 48$

$\Rightarrow \quad 2x = 48$

$\Rightarrow \quad x = 24$ m

$\therefore$ Length of the rectangle = 24 + 8 = 32 m

Breadth of the rectangle = 24 − 6 = 18 m

$\therefore$ Perimeter of the rectangle = $2(l + b)$

$= 2(32 + 18)$

$= 2(50)$ m

$= 100$

3. ***(e)*** Let the base of the right angled triangle be $4x$ and its height be $5x$.

Then, area of the right angled triangle

$= \frac{1}{2} \times 4x \times 5x$

$\Rightarrow \quad 80 = \frac{1}{2} \times 20x^2$

$\Rightarrow \quad x^2 = 8$

$\Rightarrow \quad x = 2\sqrt{2}$ cm

Height = $5x$

$= 5 \times 2\sqrt{2}$

$= 10\sqrt{2}$ cm

4. ***(a)*** In ΔBDE,

$DE = 28 + 28 = 56$ cm

$BC = 28$ cm

Area of $\Delta BDE = \frac{1}{2} \times DE \times BC$

$= \frac{1}{2} \times 56 \times 28$

$= 784 cm^2$

5. ***(b)*** Area of square = $28 \times 28 = 784 cm^2$

Area of four circles = $4\pi r^2$

$= 4 \times \frac{22}{7} \times 7 \times 7$

$= 616 cm^2$

$\therefore$ Area of shaded region = 784 − 616

$= 168 cm^2$

6. ***(c)*** Area of circle = 7 × Circumference

$\pi R^2 = 7 \times 2\pi R$

$\therefore \quad R = 14$

$\therefore$ Circumference of circle = $2\pi R$

$= 2 \times \frac{22}{7} \times 14 = 88$ units

7. ***(c)*** The number of tiles will be minimum, if size of each marble is maximum.

Size of each tile = HCF of 3.78 and 5.25m

```
378)525(1
    378
    147)378(2
        294
         84)147(1
             84
             63)84(1
                63
                21)63(3
                   63
                   ×
```

$\therefore$ HCF of 3.78 and 5.25 m = 0.21 m

$\therefore$ Number of tiles = $\frac{3.78 \times 5.25}{0.21 \times 0.21} = 450$

8. ***(b)*** Distance covered in 1 revolution

$= \pi \times$ diameter $= \pi d$

$\therefore \quad 1000 \times \frac{22}{7} \times d = 440$

$\Rightarrow \quad d = \frac{440 \times 7}{1000 \times 22} = 0.14$ m

9. ***(c)*** Let the length of the rectangle be x m and breadth be y m.

According to the question,

Case I $\quad xy - (x - 5)(y + 3) = 9$

$\Rightarrow \quad xy - (xy - 5y + 3x - 15) = 9$

$\Rightarrow \quad 5y - 3x + 15 = 9$

$\Rightarrow \quad 3x - 5y - 6 = 0 \quad \ldots(i)$

Case II $\quad (x + 3)(y + 2) - xy = 67$

$\Rightarrow \quad 3y + 2x + 6 = 67$

$\Rightarrow \quad 2x + 3y - 61 = 0 \quad \ldots(ii)$

By Eqs. (i) × 3 + (ii) × 5,

$9x - 15y - 18 = 0$

$10x + 15y - 305 = 0$

$19x = 323$

$\Rightarrow \quad x = \frac{323}{19} = 17$ m

$\therefore$ Length of the rectangle = 17 m

10. ***(e)*** Let the length of rectangle = l m

and breadth of rectangle = b m

Area of the original rectangle = lb

According to the question,

$l \times \frac{120}{100} \times b \times \frac{80}{100} = 192$

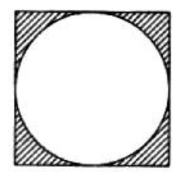
$\Rightarrow \quad 1.2l \times 0.8b = 192$

$\Rightarrow \quad lb = \frac{192}{1.2 \times 0.8}$

$\Rightarrow \quad lb = 200\ m^2$

$\therefore$ Area of the original rectangle = $200\ m^2$

11. ***(d)***



Radius of the circular garden

$= \frac{28}{2} = 14$ m

Area of the circle = πr^2

$= \frac{22}{7} \times 14 \times 14 = 616 m^2$

Area of the square plot = $28 \times 28 = 784 m^2$

$\therefore$ Area of the shaded region

$= (784 - 616) m^2 = 168 m^2$

12. ***(c)*** Side of a square = $\frac{\text{Perimeter}}{4}$

$= \frac{56}{4} = 14$ cm

$\therefore$ Smallest side of the right angled triangle

$= 14 - 8 = 6$ cm

Length of rectangle

$= \frac{\text{Area}}{\text{Breadth}} = \frac{96}{8} = 12$ cm

$\therefore$ Second side of the triangle = 12 − 4

$= 8$ cm

13. ***(b)*** Radius of circle = $\frac{28}{2} = 14$ cm

Area of circle = πr^2

$= \frac{22}{7} \times 14 \times 14 = 616 cm^2$

$\therefore$ Area of rectangle = 1166 − 616

$= 550 cm^2$

Breadth of rectangle

$= \frac{550}{25} = 22$ cm

∴ Circumference of circle

$= \pi \times \text{diameter}$

$= \frac{22}{7} \times 28 = 88$ cm

Perimeter of rectangle

= 2(length + breadth)

$= 2(25 + 22) = 94$ cm

∴ Required sum $= (88 + 94)$ cm $= 182$ cm

14. ***(a)*** For a right angled triangle,

hypotenuse $= \sqrt{6^2 + 8^2} = \sqrt{36 + 64}$

$= \sqrt{100} = 10$ cm

Largest side = 10 cm

Side of square $= 3 \times 10 = 30$ cm

∴ Diagonal of square $= \sqrt{2} \times 30$

$= 30\sqrt{2}$ cm

15. ***(b)*** Perimeter of rectangle = 668 cm

∴ $2(l + b) = 668$

⇒ $l + b = 334$

⇒ $l = (334 - b)$

Length of a rectangle

= Twice the diameter of a circle

$334 - b = 2 \times d = 2 \times 2r = 4r$

∴ $r = \frac{334 - b}{4}$

Area of square = Circumference of circle

$(22)^2 = 2\pi r$

$484 = \frac{2 \times 22(334 - b)}{7 \times 4}$

⇒ $334 - b = \frac{484 \times 7 \times 4}{2 \times 22} = 308$

⇒ $b = 334 - 308 = 26$ cm

16. ***(a)*** Perimeter of square

= 2 × Perimeter of rectangle

$= 2 \times 2(l + b) = 4(8 + 7) = 60$

Side of square $= \frac{60}{4} = 15$ cm

Diameter of semi-circle = 15 cm

∴ Circumference of semi-circle $= \frac{\pi d}{2} + d$

$= \frac{22}{7 \times 2} \times 15 + 15 = 38.57$ cm

17. ***(c)*** Side of square $= \sqrt{\text{Area}}$

$= \sqrt{196} = 14$ cm

∴ Radius of circle = 28 cm

∴ Circumference of circle

$= 2 \times \frac{22}{7} \times 28 = 176$ cm

If length of rectangle is x cm, then

$2(x + 176) = 712$

⇒ $x + 176 = \frac{712}{2} = 356$

⇒ $x = 356 - 176$

$x = 180$ cm

18. ***(a)*** Circumference of circular plot

$= \frac{7700}{14} = 550$ ft

Circumference, $2\pi r = 550$

$r = \frac{550}{2\pi} = \frac{550 \times 7}{2 \times 22}$

$r = 87.5$ ft

∴ Area of circular field $= \pi r^2$

$= \frac{22}{7} \times 87.5 \times 87.5$

= 24062.5 sq ft

19. ***(a)*** If the side of square is x cm,

then $\pi r^2 + x^2 = 2611$

⇒ $\frac{22}{7} \times 21 \times 21 + x^2 = 2611$

⇒ $1386 + x^2 = 2611$

⇒ $x^2 = 2611 - 1386$

∴ $x = 1225$

$x = \sqrt{1225} = 35$ cm

∴ Required sum of the circumference of circle + Perimeter of square

$= 2\pi r + 4x$

$= 2 \times \frac{22}{7} \times 21 + 4 \times 35$

$= 132 + 140 = 272$ cm

20. ***(d)*** Area of circular field = 32378.5

$\pi r^2 = 32378.5$

⇒ $r^2 = \frac{32378.5}{22} \times 7$

⇒ $r = \sqrt{\frac{32378.5 \times 7}{22}}$

⇒ $r = 101.5$ m

Circumference of circle $= 2\pi r$

$= 2 \times \frac{22}{7} \times 101.5$

= 638 m

∴ Expenditure of fencing

= ₹ (154 × 638)

= ₹ 98252

21. ***(a)*** Let the breadth of the rectangular plot be x m.

∴ Length $= 3x$ m

According to the question,

$3x \times x = 7803$

⇒ $x^2 = \frac{7803}{3} = 2601$

∴ $x = \sqrt{2601} = 51$ m

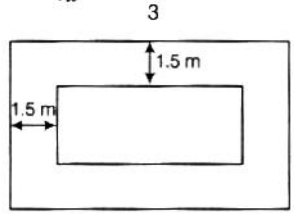
22. ***(c)*** Suppose the length of the field is x m.

So, the breadth of the field will be $\frac{3x}{4}$ m.

According to the question,

$x \times \frac{3x}{4} = 300$

⇒ $x^2 = \frac{300 \times 4}{3}$



⇒ $x^2 = 400$

⇒ $x = 20$

∴ Length = 20 m

Breadth $= \frac{3}{4} \times 20 = 15$ m

Length of the field with garden

$= 20 + 1.5 \times 2$

$= 20 + 3 = 23$ m

Breadth of the field with garden

$= 15 + 1.5 \times 2 = 18$ m

∴ Its area $= 23 \times 18 = 414\text{m}^2$

Area of garden $= (414 - 300)\text{m}^2$

$= 114\text{m}^2$

23. ***(c)*** Let the length of rectangular plot be x m and breadth be y m.

∴ Perimeter $= 2(x + y) = 340$ m

Now, area of the boundary

$= (x + 2)(y + 2) - xy$

$= xy + 2x + 2y + 4 - xy$

$= 2x + 2y + 4$

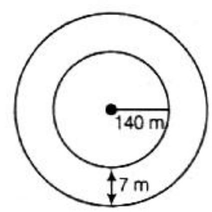
$= 2(x + y) + 4$

$= 340 + 34 = 344$

∴ Cost of gardening $= 344 \times 10$

= ₹ 3440

24. ***(e)*** Radius of the field = 140 m



Width of the garden = 7 m

∴ Area of garden $= \pi(147^2 - 140^2)$

$= \frac{22}{7}(147 + 140)(147 - 140)$

$= \frac{22}{7} \times 287 \times 7$

$= 22 \times 287 = 6314\text{m}^2$

∴ Required cost = ₹ 21 × 6314

= ₹ 132594

25. ***(a)*** $D_1 = 2$ cm, $r_1 = 1$ cm,

$D_2 = 4$ cm,

$r_2 = 2$ cm

∴ Volume of the silver used in the ball

$= \frac{4}{3}\pi(r_2^3 - r_1^3)$

$= \frac{4}{3}\pi[(2)^3 - (1)^3]$

$= \frac{4}{3} \times \frac{22}{7} \times 7$

$= \frac{28}{3}\pi\text{ cm}^3$

26. ***(c)*** Let the length, breadth and height of the cuboid be a, b and c cm respectively.

$$2(ab + bc + ca) = 22$$

and $\quad 4(a + b + c) = 24$

$\Rightarrow \quad a + b + c = 6$

$\Rightarrow (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$

$\Rightarrow \quad 36 = a^2 + b^2 + c^2 + 22$

$\Rightarrow \quad a^2 + b^2 + c^2 = 14$

$\Rightarrow \quad \sqrt{a^2 + b^2 + c^2} = \sqrt{14}$

= Diagonal of cuboid

27. ***(a)*** Volume of the cylindrical jar $= \pi r_1^2 h$ ...(i)

Now, volume of the cylindrical jar on reducing height $= \pi r_2^2 \dfrac{64}{100} h$

$$= \frac{16}{25} \pi r_2^2 h \qquad \text{... (ii)}$$

According to the question,

$$\pi r_1^2 h = \frac{16}{25} \pi r_2^2 h$$

$$\frac{r_2^2}{r_1^2} = \frac{25}{16}, \left(\frac{r_2}{r_1}\right) = \frac{5}{4}$$

$\therefore$ Increased radius

$$\left(\frac{r_2 - r_1}{r_1}\right) = \left(\frac{5 - 4}{4}\right) \times 100$$

$$= \frac{1}{4} \times 100 = 25\%$$

So, radius must be increased by 25%.

28. ***(b)*** According to the question,

in both cases, volume will be the same.

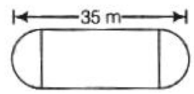
$\therefore \quad \dfrac{4}{3} \pi r^3 = \pi R^2 H$

$\Rightarrow \quad \dfrac{4}{3} \pi r^3 = \pi R^2 \cdot 2r \qquad [\because H = 2r]$

$\Rightarrow \quad \dfrac{2}{3} r^2 = R^2$

$\therefore \quad R = r\sqrt{\dfrac{2}{3}}$

29. ***(d)*** Let the radius of the cylinder be r and height of the cylinder be h cm.



Then, $\quad r = \dfrac{h}{8}$

and $\quad h = 35 - 2r$

$\Rightarrow \quad h = 35 - 2 \times \dfrac{h}{8}$

$\Rightarrow \quad h = 35 - \dfrac{h}{4}$

$\Rightarrow \quad \dfrac{5h}{4} = 35$

$\Rightarrow \quad h = \dfrac{35 \times 4}{5} = 28\text{ cm}$

Now, $\quad r = \dfrac{h}{8} = \dfrac{28}{8} = \dfrac{7}{2}\text{ cm}$

$\therefore$ Radius of hemisphere $= \dfrac{7}{2}$ cm

Total surface area of solid

$$= (2\pi rh + 2 \times 2\pi r^2)\text{cm}^2$$

$$= 2\pi r(h + 2r)\text{cm}^2$$

$$= 2 \times \frac{22}{7} \times \frac{7}{2}\left(28 + 2 \times \frac{7}{2}\right)\text{cm}^2$$

$$= 2 \times \frac{22}{7} \times \frac{7}{2} \times 35\text{cm}^2$$

$$= 770\text{cm}^2$$

30. ***(b)*** Volume of earth dug out

$$= \frac{22}{7} \times 7 \times 7 \times 10\text{m}^3$$

$$= 1540\text{m}^3$$

Area of embankment

$$= \frac{22}{7}[(28)^2 - (7)^2]\text{m}^2$$

$$= \frac{22}{7} \times (28 + 7)(28 - 7)\text{m}^2$$

$$= \frac{22}{7} \times 35 \times 21\text{m}^2$$

$$= 2310\text{m}^2$$

$\therefore$ Height of embankment

$$= \frac{\text{Volume}}{\text{Area}} = \frac{1540}{2310}$$

$$= \frac{2}{3}\text{ m}$$

31. **(b)**

32. **(c)**