Special

GEOMETRY + MENSURATION TEST FOR SSC SOLUTION

By – Alok Sir

1. (A) The angle subtended at the centre by an are is twice to that of angle subtended at the circumference.

A

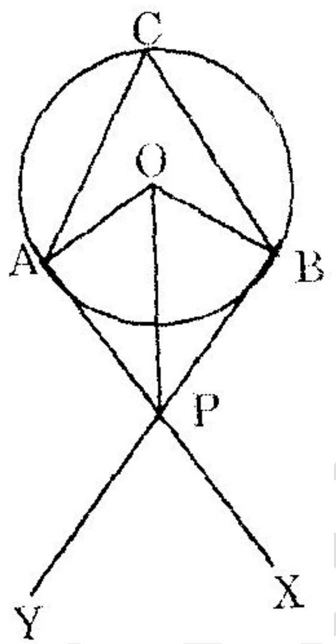
$\therefore \angle CAD = \frac{1}{2}\angle COD = 70°$

$\therefore \angle BAD = 70° + 40° = 110°$

$\therefore \angle BCD = 180° - 110° = 70°$

2. (B)

3. (A)



$\angle ACB = 65°$

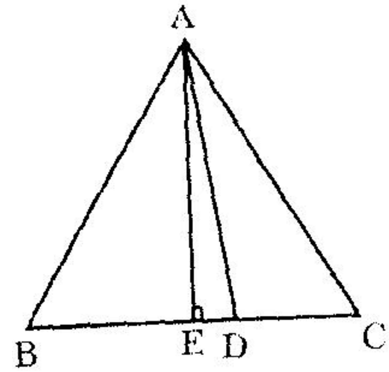
$\angle AOB = 2 \times 65° = 130°$

$\angle OAP = 90°; \angle AOP = 65°$

$\therefore \angle APO = 180° - 90° - 65°$

$= 25°$

4. (B)



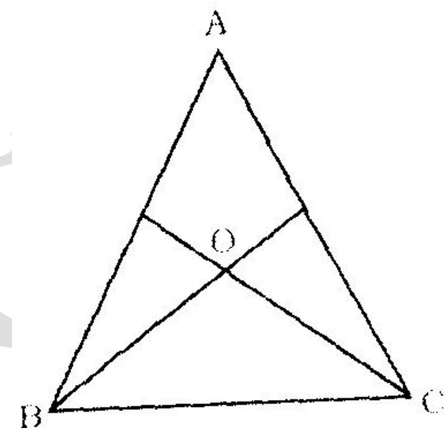
$\angle A = 180° - 60° - 40° = 80°$

$\angle BAD = \frac{80}{2} = 40°$

$\angle BAE = 180° - 60° - 90° = 30°$

$\therefore \angle DAE = 40° - 30° = 10°$

5. (A)



In

In ΔABC

$\angle A + \angle B + \angle C = 180°$

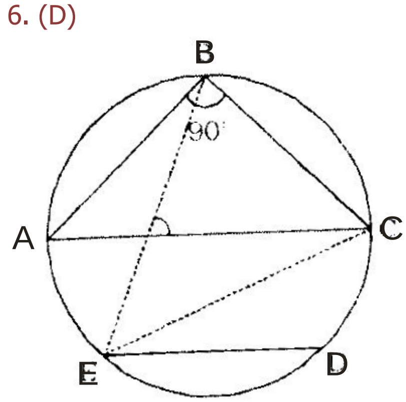
In $\Delta BOC \angle BOC = 110°$

$\therefore \frac{B}{2} + \frac{C}{2} = 180° - 110° = 70°$

$\Rightarrow B + C = 140°$

$\therefore \angle BAC = 180° - 140° = 40°$

6. (D)



$\angle CBE = 50°$

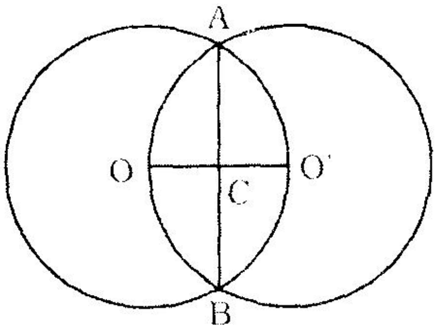
$\angle BAC + BCA = 90°$

$\angle ABE = 90° - 50° = 40°$

$\therefore \angle ABE = \angle ACE = 40°$

$\therefore \angle ACE = \angle DEC = 40°$

7. (B)



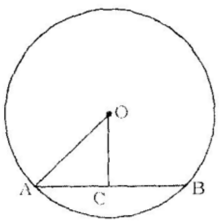
OC = 2cm
OA = 4cm

$\therefore AC = \sqrt{4^2 - 2^2} = \sqrt{16-4}$

$= \sqrt{12} = 2\sqrt{3}$

$\therefore AB = 4\sqrt{3}$cm

8. (B)



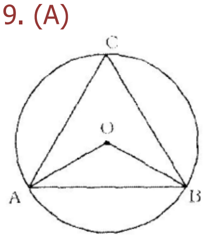
AC = CB = 4cm
OC = 3 cm

$\therefore OA = \sqrt{OC^2 + CA^2}$

$= \sqrt{3^2 + 4^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$cm

9. (A)



AO = OB = AB

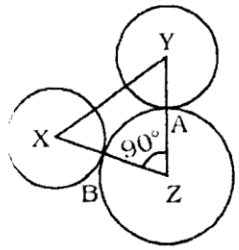
$\therefore \angle AOB = 60°$

$\therefore \angle ACB = 30°$

10. (A)

11. (b)

$XZ = r + 9$



$YZ = r + 2$

$\therefore XY^2 = XZ^2 + ZY^2$

$\Rightarrow 17^2 = (r + 9)^2 + (r + 2)^2$

$\Rightarrow 289 = r^2 + 18r + 81 + r^2 + 4r + 4$

$\Rightarrow 2r^2 + 22r + 85 - 289 = 0$

$\Rightarrow 2r^2 + 22r - 204 = 0$

$\Rightarrow r^2 + 11r - 102 = 0$

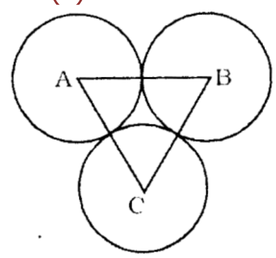
$\Rightarrow r^2 + 17r - 6r - 102 = 0$

$\Rightarrow r(r + 17) - 6(r + 17) = 0$

$\Rightarrow (r - 6)(r + 17) = 0$

$\Rightarrow r = 6$ cm

12. (d)



AB = 4 + 6 = 10 cm
BC = 6 + 8 = 14 cm
CA = 8 + 4 = 12 cm

$\therefore$ Semi-perimeter(s)

$= \frac{10 + 14 + 12}{2}$

= 18 cm

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

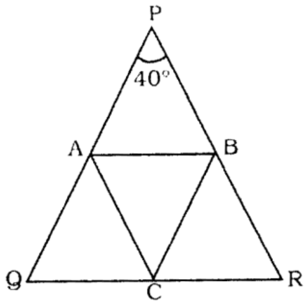
$= \sqrt{18(18-10)(18-14)(18-12)}$

$= \sqrt{18 \times 8 \times 4 \times 6}$

$= 3 \times 2 \times 2 \times 2\sqrt{6}$

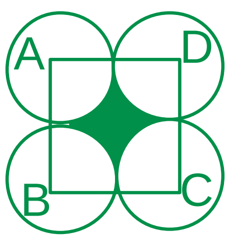
$= 24\sqrt{6}$ sq.cm.

13. (d)



AC = QC
$\therefore \angle QAC = \angle CQA = x$
CR = CB
$\therefore \angle CBR = \angle CRB = y$
$\therefore$ From ΔPQR,
$\angle x + \angle y + 40° = 180$
$\angle x + \angle y = 140°$ ......(i)
Again,
$\angle ACQ + \angle ACB + \angle BCR = 180°$
$\Rightarrow 180° - 2x + \angle ACB + 180° - 2y = 180°$
$\Rightarrow \angle ACB = 2(x + y) - 180°$
$= 2 \times 140 - 180° = 100°$

14. (b) Area of the shaded region = Area of square of side 6cm-4×a right angled sector



$= 36 - 4 \times \frac{\pi \times 3^2}{4}$

$= 36 - 9\pi = 9(4 - \pi)$ sq. cm.

15. (a) $\pi(r+1)^2 - \pi r^2 = 22$

$\Rightarrow \pi(r^2 + 2r + 1 - r^2) = 22$

$\Rightarrow 2\pi r + \pi = 22$

$\Rightarrow \frac{22}{7}(2r+1) = 22$

$\Rightarrow 2r + 1 = 7$

$\Rightarrow 2r = 6 \Rightarrow r = 3$ cm.

16. (a) Volume of earth : volume of moon

$= \frac{4}{3}\pi r^3 : \frac{4}{3}\pi\left(\frac{r}{4}\right)^3 = 64:1$

17. (d) $\pi r + 2r = 36$

$\Rightarrow r\left(\frac{22}{7} + 2\right) = 36$

$\Rightarrow r\left(\frac{22}{7} + 2\right) = 36$

$\Rightarrow r\left(\frac{22+14}{7}\right) = 36$

$\Rightarrow r = \frac{36 \times 7}{36} = 7$ meter

18. (a) $\pi r_1^2 : \pi r_2^2 = 4:7$

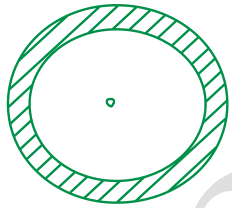
$\Rightarrow r_1 : r_2 = \sqrt{4} : \sqrt{7} = 2 : \sqrt{7}$

19. (C) Area of circle $= kr^2$

Area of shaded region $= k(5^2 - 3^2) = 16\pi$ sq.cm.

Area of larger circle $k \times 5^2 = 25\pi$ sq. units

So that rquired ratio =16:25



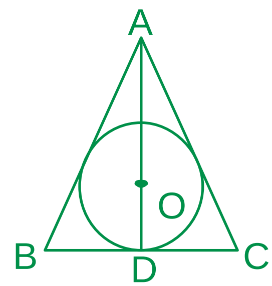
20. (c) The diameter of the greatest circle inscribed inside a squae will be equal to the side of square i.e., 21 cm.
So radius of the circle =21/2
So area of the circle = $= \pi \times (\text{radius})^2$

$= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = \frac{693}{2}$ cm^2

21. (b) Let ABC be the equilateral triangle o side 42 cm and let AD be perpendicular from A on BC since the triangle is equilateral, so D bisects BC
So BD=CD=21 cm.
The centre of the inscribed circule will coincide with the centroid of Δ ABC

Therefore, OD= $\frac{1}{3}$AD

In ΔABC



$AB^2 = AD^2 + BD^2$

$\Rightarrow 42^2 = AD^2 + 21^2$

$\Rightarrow AD = \sqrt{42^2 - 21^2}$

$= \sqrt{(42+21)(42-1)}$

$= \sqrt{63 \times 21} = 3 \times 7\sqrt{3}$ cm.

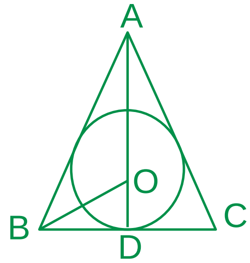
So area of the incircle =

$\pi(OD)^2$

$= \frac{22}{7} \times 7\sqrt{3} \times 7\sqrt{3}$

$= 22 \times 7 \times 3 = 462$ cm^2

22. (c)Let the each side of the equilateral triangel be 2x cm.
BD=x.



Radius of incircle =OD= $\frac{1}{3}$AD

$= \frac{1}{3}\sqrt{(2x)^2 - x^2}$

$= \frac{\sqrt{3}x}{3} = \frac{x}{\sqrt{3}}$ cm

According to the question

$\pi\left(\frac{2x}{\sqrt{3}}\right)^2 - \pi\left(\frac{x}{\sqrt{3}}\right)^2 = 44$

$\Rightarrow \frac{4\pi x^2}{3} - \frac{\pi x^2}{3} = 44$

$\Rightarrow \pi x^2 = 44 \Rightarrow x^2 = \frac{44 \times 7}{22} = 14$

Area of the equilateral triangle

$= \frac{\sqrt{3}}{4} \times (\text{side})^2 = \frac{\sqrt{3}}{4} \times (2x)^2$

$= \sqrt{3}x^2 = 14\sqrt{3}$ sq.cm.

23. (c) When the rectangular sheet is rolled along its length, the length of the sheet forms the circumference of the base of cylidner and breadth of sheet forms the height of cylinder.

Circumference =1006

$\Rightarrow 2\pi r = 100$

$\Rightarrow 2 \times \frac{22}{7} \times r = 100$

$\Rightarrow r = \frac{700}{44} = \frac{175}{11}$ cm

So that volume of the cylinder = $\pi r^2 h$

$= \frac{22}{7} \times \frac{175}{11} \times \frac{175}{11} \times 44 = \frac{245000}{7} = 35000 \text{ cm}^3$

24. (d) If the radius of the hemisphere be r units, then hight of cylinder and cone =r units

So that required ratio =

$\Rightarrow \pi r^2 h : 2\pi r^3 : \frac{1}{3}\pi r^2 h$

$= \pi r^3 : 2\pi r^3 : \frac{1}{3}\pi r^3$

$= 3 : 6 : 1$

25. (d) Let the radius of the base of cylinder be r units.

Height =8 r units

Its volume $= \pi r^2 \times 8r \Rightarrow 8\pi r^3$ cu. units

Radius of sphere = r/2 units

Volume $= \frac{4}{3}\pi\left(\frac{4}{2}\right)^3 = \frac{\pi r^3}{6}$ cu. units

So that Number of spherical balls $= \frac{8\pi r^3}{\pi r^3} \times 6 = 48$

26. (a) Slant height of cone (l)

$= \sqrt{r^2 + h^2} = \sqrt{r^2 + r^2} = \sqrt{2}r$

=Total surface area of cone/total surface area of hemi sphere

$= \frac{\pi r l + \pi r^2}{3\pi r^2} = \frac{l + r}{3r} = \frac{\sqrt{2}r + r}{3r} = \sqrt{2} + 1 : 3$

27. (d) volume of sphere =

$\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 9 \times 9 \times 9$

$= 972\pi$ cu.cm.

If the radius of wire be R cm. Then

$\pi R^2 \times 10800 = 972\pi$

$\Rightarrow R^2 = \frac{972}{10800} = 0.09$

$\therefore R = \sqrt{0.09} = 0.03$ cm

Diameter $= 2 \times 0.3 = 0.6$ cm.

28. (a) Volume of metallic cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 30 \times 30 \times 45$ cu.cm.

volume of sphere $= \frac{4}{3}\pi R^3$

$= \frac{4}{3}\pi \times 5 \times 5 \times 5$ cu.cm.

So that Required number of spheres

$= \frac{\frac{1}{3}\pi \times 30 \times 30 \times 45}{\frac{4}{3}\pi \times 5 \times 5 \times 5} = 81$

29. (d)

(4) Sum of interior angles

$= (2n - 4) \times 90°$

Sum of exterior angles = 360°

$\therefore (2n - 4) \times 90° = 360° \times 2$

$\Rightarrow 2n - 4 = 2 \times 360° \div 90 = 8$

$\Rightarrow 2n - 4 = 8 \Rightarrow 2n = 12 \Rightarrow n = 6$

30. (b)

(2) Each interior angle

$= \left(\frac{2n-4}{n}\right) \times 90°$

$\therefore \frac{(2n-4) \times 90°}{n} = 105°$

$\Rightarrow (12n-4) \times 6 = 7n$
$\Rightarrow 12n - 24 = 7n$
$\Rightarrow 5n = 24$

$\Rightarrow n = \frac{24}{5}$ which is impossible.

31. (c) Let the number of sides be 5x and 6x respectively.

$\therefore \frac{\frac{10x-4}{5x}}{\frac{12x-4}{6x}} = \frac{24}{25}$

$\left[\text{Each interior angle} = \frac{(2n-4)90°}{n}\right]$

$\Rightarrow \frac{5x-2}{5} \times \frac{6}{6x-2} = \frac{24}{25}$

$\Rightarrow \frac{5x-2}{6x-2} = \frac{4}{5}$

$\Rightarrow 25x - 10 = 24x - 8$
$\Rightarrow x = 10 - 8 = 2$
$\therefore$ Number of sides = 10 and 12.

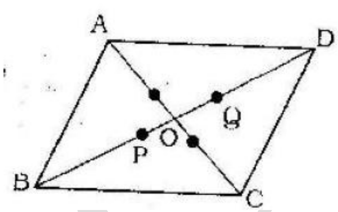
32.(b)

(2) Let the number of sides of regular polygon be n.

$\therefore \left(\frac{2n-4}{n}\right) \times 90° = 2 \times \frac{360}{n}$

$\Rightarrow (2n-4) = 8$
$\Rightarrow 2n = 12$
$\Rightarrow n = 6$

33. (b)



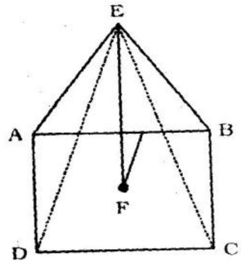
Centroid is the point where medians intersect. Diagnals of parallelogram bisect each other.

$OP = \frac{1}{3} \times 9 = 3$ cm

$OQ = \frac{1}{3} \times 9 = 3$cm

$\therefore PQ = 6$cm

34. (B)



Height of the triangle

$= \sqrt{15^2 + 8^2}$

$= \sqrt{225 + 64} = \sqrt{289}$

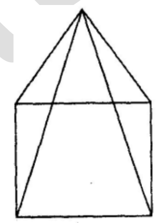
$= 17$ cm

$\therefore$ Area of the lateral surface of pyramid = 4 × Area of triangle

$\therefore \quad = 4 \times \frac{1}{2} \times$ base × height

$= 4 \times \frac{1}{2} \times 16 \times 17 = 544$ sq.cm.

35. (B) Lateral surface area



$= \frac{1}{2} \times$ perimeter of base × slant height

$\therefore$ Slant height $= \sqrt{8^2 + 15^2}$

$= \sqrt{64 + 225}$

$= \sqrt{289} = 17$ cm

$\therefore$ Required area $= \frac{1}{2} \times 64 \times 17$

= 544 sq.cm.

36. (D),

37. (C) The distance travelled by wheel in one revolution

$= 2\pi r = 2 \times \frac{22}{7} \times 1.75\,\text{m} = 11$ m

Therefore, the number of revolution to cover 11 km i.e. 11000 m by wheel

$= \frac{11000}{11} = 1000$

38. (B) The distance covered

= 2 km 26 decameters

= (2 × 1000 + 26 × 10) m.

= 2260 m.

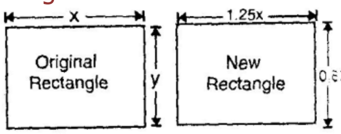
The distance covered in one revolution

$= \frac{\text{Total distance}}{\text{Number of revolutions}}$

$= \frac{2260}{113} = 20\,\text{m}.$

Obviously,
Circumference of wheel
= 20 m
or, p × diameter = 20 m.
or, Diameter $= \frac{20}{\pi} = \frac{20\times 7}{22} = \frac{70}{11} = 6\frac{4}{11}$ m.

39. (C)
40. (C) ∴ According to question,
Area of new rectangle



= xy, area of new rectangle =xy.
= 1.25x × 0.80y= xy
or Effective change

$$= \left(25 - 20 - \frac{25\times 20}{100}\right)\% = 0\%$$

41. (C) Required percentage decrease

$$= 100 - \frac{50\times 50\times 150}{100\times 100}$$
$$= 100 - 37.5 = 62.5\%$$

42.(c) The equation of line passing through (2,-3) and parallel to y-axis is (Y+3)= tan 90 (x-2)

43.(b) Let the point of foot of the perpendicular be M(x_1, y_1)
Now, $PM \perp AB$
Slope of line AB is $m_1 = -1$
and slope of line MP is

$$m_2 = \frac{y_1 - 3}{x_1 - 2}$$
$$\Rightarrow \quad m_1 m_2 = -1$$
$$\Rightarrow \quad -1\left(\frac{y_1 - 3}{x_1 - 2}\right) = -1$$
$$\Rightarrow \quad y_1 - 3 = x_1 - 2 \Rightarrow x_1 - y_1 + 1 = 0 \quad \text{....(i)}$$

Since, the point M lies on the line AB, then
$x_1 + y_1 - 11 = 0$
On solving Eqs. (i) and (ii) we get
$2x_1 - 10 = 0 \Rightarrow x_1 = 5$ and $y_1 = 6$
So, required foot of the perpendicular M is (5,6).

44. (a) If two lines $a_2 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$ are parallel,
then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
Then, for the lines x+2y –9=0 and kx+4y+5 =0

$$\frac{1}{k} = \frac{2}{4} = -\frac{9}{5}$$

Taking the first two parts, K= 2

45.(b) Given that, the vertices of the triangle are $A(-4,2), B(0,-1)$ and $C(3,3)$

∴ Length of AB

$$= \sqrt{(-4-0)^2 + (2+1)^2} = \sqrt{16\times 9} = \sqrt{25} = 5$$

Length of BC

$$= \sqrt{(0-3)^2 + (-1-3)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

Length of CA

$$= \sqrt{(3+4)^2 + (3-2)^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$$

∴ Perimeter of Δ =(AB+BC+CA)

$$= \left(5+5+5\sqrt{2}\right) = \left(10+5\sqrt{2}\right)$$