Test Series 1

NATIONAL DEFENCE ACADEMY

SOLUTION (HINDI+ENGLISH)

ANSWERSHEET

1.	(b)	2.	(d)	3.	(b)	4.	(d)	5.	(a)	6.	(b)	7.	(c)	8.	(a)	9.	(c)	10.	(c)
11.	(b)	12.	(c)	13.	(c)	14.	(c)	15.	(b)	16.	(d)	17.	(b)	18.	(d)	19.	(a)	20.	(b)
21.	(a)	22.	(c)	23.	(a)	24.	(c)	25.	(b)	26.	(a)	27.	(d)	28.	(c)	29.	(b)	30.	(b)
31.	(b)	32.	(b)	33.	(b)	34.	(b)	35.	(a)	36.	(c)	37.	(a)	38.	(a)	39.	(d)	40.	(a)
41.	(d)	42.	(a)	43.	(b)	44.	(b)	45.	(c)	46.	(d)	47.	(c)	48.	(a)	49.	(d)	50.	(d)
51.	(d)	52.	(a)	53.	(b)	54.	(c)	55.	(c)	56.	(b)	57.	(a)	58.	(c)	59.	(b)	60.	(d)
61.	(a)	62.	(c)	63.	(c)	64.	(a)	65.	(b)	66.	(a)	67.	(a)	68.	(d)	69.	(d)	70.	(a)
71.	(c)	72.	(b)	73.	(b)	74.	(b)	75.	(a)	76.	(d)	77.	(a)	78.	(d)	79.	(d)	80.	(a)
81.	(d)	82.	(b)	83.	(c)	84.	(b)	85.	(b)	86.	(d)	87.	(d)	88.	(c)	89.	(b)	90.	(c)
91.	(a)	92.	(a)	93.	(c)	94.	(c)	95.	(c)	96.	(a)	97.	(b)	98.	(d)	99.	(b)	100.	(b)
101.	(c)	102.	(d)	103.	(a)	104.	(b)	105.	(d)	106.	(c)	107.	(b)	108.	(d)	109.	(d)	110.	(d)
111.	(b)	112.	(a)	113.	(c)	114.	(b)	115.	(b)	116.	(b)	117.	(d)	118.	(b)	119.	(d)	120.	(c)

SOLUTION

1. Given $(1+3+5+\dots+p)+(1+3+5+\dots+q)$
$=(1+3+5+\dots+r)$

Now $p=1+(n-1)2 \Rightarrow n=\dfrac{p+1}{2}$

$[\because T_n=a+(n-1)d]$

Hence $\dfrac{\frac{p+1}{2}}{2}\left[2\times1+\left(\dfrac{p+1}{2}-1\right)2\right]+\dfrac{\frac{q+1}{2}}{2}\times\left[2\times1+\left(\dfrac{q+1}{2}-1\right)2\right]$

$=\dfrac{\frac{r+1}{2}}{2}\left[2\times1+\left(\dfrac{r+1}{2}-1\right)2\right]$

$\left[\because S_n=\dfrac{n}{2}\{2a+(n-1)d\}\right]$

$\Rightarrow \dfrac{p+1}{4}[2+(p-1)]+\dfrac{q+1}{4}[2+(q-1)]$

$=\dfrac{r+1}{4}[2+r-1]$

$\Rightarrow (p+1)^2+(q+1)^2=(r+1)^2$

which is possible only when $p=7, q=5$ and $r=9$.

Hence $p+q+r=7+5+9=21$.

2. We have $A=\{x \mid x\le 9, x\in N\}$
$=\{1,2,3,4,5,6,7,8,9\}$

Total possible multiple of 3 are

3, 6, 9, 12, 15, 18, 21, 24, 27

Here 3 and 27 are not possible.

$\therefore$ $6=1+2+3$

$9=2+3+4, 5+3+1, 6+2+1$

$12=9+2+1, 8+3+1, 7+1+4, 7+2+3$

$6+4+2, 6+5+1, 5+4+3$

$15=9+4+2, 9+5+1, 8+6+1, 8+5+2$

$8+4+3, 7+6+2, 7+5+1+3; 6+5+4$

$18=9+8+1, 9+7+2, 9+6+3,$

$9+5+4, 8+7+3, 8+6+4$

$7+6+5$

$21=9+8+4, 9+7+5, 8+7+6$

$24=9+8+7$

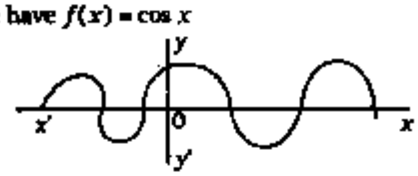
$\therefore$ Largest possible number of subsets like B.

3. Here $A=\{-1,2,5,8\}$ and $B=\{0,1,3,6,7\}=30$.

Then $R=\{(-1,0),(2,3),(5,6)\}$

$\therefore$ R will contain 3 elements.

4. We have $f(x)=\cos x$



From figure we can say that $f(x)$ is neither one-one nor onto.

5. Given α is a complex number such that $\alpha^2+\alpha+1=0$ then $\alpha=\omega$ or ω^2.

Hence $\alpha^{31}=(\omega)^{31}=(\omega)^{3\times10+1}=(\omega^3)^{10}.\omega$

$=1.\omega=\omega=\alpha$

6. $\because x^2, y^2$ and z^2 are in AP

$\therefore 2y^2=x^2+z^2$

Now checking the options

Here (b) Let $y+z, z+x, x+y$ are in HP

or $\dfrac{1}{y+z}, \dfrac{1}{z+x}, \dfrac{1}{x+y}$ are in AP

$\Rightarrow \quad \frac{2}{z+x} = \frac{1}{y+z} + \frac{1}{x+y}$

$\Rightarrow \quad \frac{2}{(z+x)} = \frac{x+y+y+z}{(x+y)(y+z)}$

$\Rightarrow \quad 2yz + z^2 + zx + 2xy + xz + x^2$

$= 2yx + 2y^2 + 2zx + 2yz$

$\Rightarrow \quad z^2 + x^2 = 2y^2$

$\Rightarrow \quad x^2, y^2, z^2$ are in A.P.

Hence, $y+z, z+x, x+y$ are in A.P.

7. 11th term of the given group = (56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66)

$\therefore$ Required sum

$= 56 + 57 + 58 + 59 + 60 + 61 + 62 + 63 + 64 + 65 + 66$

$= 671$

8. α, β are roots of $ax^2 + bx + b = 0$

$\therefore \quad \alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{b}{a}$

Now $\sqrt{\frac{\alpha}{\beta}} + \frac{\sqrt{\beta}}{\sqrt{\alpha}} + \frac{\sqrt{b}}{\sqrt{a}} = \frac{\alpha+\beta}{\sqrt{\alpha\beta}} + \frac{\sqrt{b}}{\sqrt{a}}$

$= \frac{-\frac{b}{a}}{\sqrt{\frac{b}{a}}} + \frac{\sqrt{b}}{\sqrt{a}}$

$= -\frac{\sqrt{b}}{\sqrt{a}} + \frac{\sqrt{b}}{\sqrt{a}} = 0$

9. $\because \sin\alpha, \cos\alpha$ are the roots of $ax^2 + bx + c = 0$

$\therefore \quad \sin\alpha + \cos\alpha = -\frac{b}{a} \quad$ ...(i)

$\sin\alpha\cos\alpha = \frac{c}{a} \quad$ ...(ii)

Squaring both sides of eqn. (i), we get

$\sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha = \frac{b^2}{a^2}$

$\Rightarrow \quad 1 + 2\sin\alpha\cos\alpha = \frac{b^2}{a^2}$

$\Rightarrow \quad 1 + 2\frac{c}{a} = \frac{b^2}{a^2}$

$\Rightarrow \quad a^2 + 2ca = b^2$

$\Rightarrow \quad b^2 - a^2 = 2ac$

10. $(1 - 2x + 3x^2 + \ldots)^{1/2} = \left[\frac{1}{(1-x)} + \frac{x}{(1-x)^2}\right]^{1/2}$

$= ((1-x)^{-2})^{\frac{1}{2}}$

$= (1-x)^{-1}$

$= 1 + x + x^2 + x^3 + x^4 + \ldots$

$\therefore$ Coefficient of x^4 is 1.

11. We have $x = 2 + 2^{\frac{1}{3}} + 2^{2/3}$

$\Rightarrow \quad x - 2 = 2^{1/3}(1 + 2^{1/3})$

$\Rightarrow \quad (x-2)^3 = [2^{1/3}(1+2^{1/3})]^3$

$\Rightarrow \quad x^3 - 6x^2 + 12x - 8$

$= 2(1 + 2 + 3.2^{1/3} + 3.2^{2/3})$

$\Rightarrow \quad x^3 - 6x^2 + 6x = 14 + 6.2^{1/3} + 62^{2/3} - 6x$

$= 14 + 6.2^{1/3} + 6.2^{2/3} - 6(2 + 2^{1/3} + 2^{2/3})$

$= 2$

12. Here $\frac{(\log_{27} 9)(\log_{16} 64)}{\log_4 \sqrt{2}} = \frac{\log 3^3 (3^2) . \log 4^2 (4)^3}{\log 2_2 (2^{1/2})}$

$= \frac{\frac{2}{3}\log_3 3 \times \frac{3}{2}\log_4 4}{\frac{1}{2\times 2}\log_2 2} = \frac{1}{\frac{1}{4}} = 4$

13. $\because \quad 2X - 3Y = \begin{bmatrix} -7 & 0 \\ 7 & -13 \end{bmatrix} \quad$ ...(i)

and $\quad 3X + 2Y = \begin{bmatrix} 9 & 13 \\ 4 & 13 \end{bmatrix} \quad$ ...(ii)

Now multiplying (i) by 3 and (ii) by 2 and subtracting (i) from (iii),

$13Y = 2\begin{bmatrix} 9 & 13 \\ 4 & 13 \end{bmatrix} - 3\begin{bmatrix} -7 & 0 \\ 7 & 13 \end{bmatrix}$

$\Rightarrow \quad 13Y = \begin{bmatrix} 39 & 26 \\ -13 & 65 \end{bmatrix}$

$\Rightarrow \quad Y = \begin{bmatrix} 3 & 2 \\ -1 & 5 \end{bmatrix}$

14. Given a, b, c are non-zero real numbers and

$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = 0$

$\therefore$ Changing $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we get

$\begin{vmatrix} 1+a & -a & -a \\ 1 & b & 0 \\ 1 & 0 & c \end{vmatrix} = 0$

$\Rightarrow \quad 1(ab) + c(b + ab + a) = 0$

$\Rightarrow \quad ab + bc + abc + ac = 0$

$\Rightarrow \quad ab + bc + ac = -abc$

$\Rightarrow \quad \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = -1$

15. Given $A' = A$ and $A' = -A$

Using both, we get $A = -A$

$\Rightarrow A = 0$

16. Here $A = \begin{bmatrix} 1 & -2 & -3 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{bmatrix}$

Clearly its diagonal elements are not zero.

so it is not antisymmetric matrix.

Now $|A| = 1(1+4) + 2(2+6) - 3(4-3)$

$= 5 + 16 - 3 = 18 \neq 0$

$\Rightarrow A$ is non singular matrix.

17. Here $A = \begin{vmatrix} 2a & 3r & x \\ 4b & 6s & 2y \\ -2c & -3t & -z \end{vmatrix} = \lambda \begin{vmatrix} a & r & x \\ b & s & y \\ c & t & z \end{vmatrix}$

$\Rightarrow \quad 2 \times 2 \times 3 \begin{vmatrix} a & r & x \\ b & s & y \\ -c & -t & -z \end{vmatrix} = \lambda \begin{vmatrix} a & r & x \\ b & s & y \\ c & t & z \end{vmatrix}$

$\Rightarrow \quad -12 \begin{vmatrix} a & r & x \\ b & s & y \\ c & t & z \end{vmatrix} = \lambda \begin{vmatrix} a & r & s \\ b & s & y \\ c & t & z \end{vmatrix}$

$\Rightarrow \quad \lambda = -12$

18. Let $A = \begin{vmatrix} 1-i & \omega^2 & \omega \\ \omega^2 + i & \omega & -i \\ 1-2i-\omega^2 & \omega^2 - \omega & i-\omega \end{vmatrix}$

Now operating $R_3 \to R_1 - R_2 - R_3$

$\therefore \quad A = \begin{vmatrix} 1 & i & \omega^2 & -\omega \\ \omega^2 + i & \omega & -i \\ 0 & 0 & 0 \end{vmatrix}$

$= 0$

19. Here, length of arc of a circle $= 2\pi r \dfrac{\theta}{360}$

Here $r = 5$ cm $\theta = 15°$

$= \dfrac{2\pi \times 5 \times 15}{360}$

$= \dfrac{5\pi}{12}$ cm

20. Let $f(\theta) = \sin\theta \cos\theta$

$= \dfrac{1}{2} 2 \sin\theta \cos\theta$

$= \dfrac{\sin 2\theta}{2}$

$\because$ Maximum value of $\sin 2\theta$ is 1.

$\therefore \quad f(\theta) = \dfrac{1}{2}$

21. Here, given $\sin x + \operatorname{cosec} x = 2$

Now $\sin^4 x + \operatorname{cosec}^4 x = (\sin^2 x + \operatorname{cosec}^2 x)^2 - 2$

$= [(\sin x + \operatorname{cosec} x)^2 - 2]^2 - 2$

$= [2^2 - 2]^2 - 2$

$= (2)^2 - 2$

$= 2$

22. $\tan 15° + \cot 15° = \dfrac{\sin 15}{\cos 15} + \dfrac{\cos 15}{\sin 15}$

$= \dfrac{\sin^2 15 + \cos^2 15}{\sin 15 \cos 15}$

$= \dfrac{1.2}{2 \sin 15 \cos 15}$

$= \dfrac{2}{\sin 30} = \dfrac{2}{\frac{1}{2}} = 4$

23. Here $\quad A + B + C = \pi/2 \quad \ldots(i)$

$\therefore \quad \tan(A + B + C) = \tan \pi/2$

$\Rightarrow \quad \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A} = \infty = \dfrac{1}{0}$

$\Rightarrow \quad 1 - \tan A \tan B + \tan B \tan C + \tan C \tan A = 0$

$\Rightarrow \quad \tan A \tan B + \tan B \tan C + \tan C . \tan A = 1$

24. Let the angle of a triangle be t, $2t$, and $3t$

$\therefore \quad t + 2t + 3t = 180$

$\Rightarrow \quad 6t = 180° \quad \Rightarrow t = 30°$

Now using sine rule, we get

$\dfrac{a}{\sin 30} = \dfrac{b}{\sin 60} = \dfrac{c}{\sin 90}$

$\Rightarrow \quad \dfrac{a}{\frac{1}{2}} = \dfrac{b}{\frac{\sqrt{3}}{2}} = \dfrac{c}{1}$

$\Rightarrow \quad \dfrac{a}{1} = \dfrac{b}{\sqrt{3}} = \dfrac{c}{2}$

$\Rightarrow \quad a : b : c = 1 : \sqrt{3} : 2$

25. Given $c(n, 12) = c(n, 8)$

or ${}^nC_{12} = {}^nC_8$

$\Rightarrow \quad \dfrac{\lfloor n}{\lfloor 12 \lfloor n-12} = \dfrac{\lfloor n}{\lfloor 8 \lfloor n-8} \Rightarrow \dfrac{\lfloor n-8}{\lfloor n-12} = \dfrac{\lfloor 12}{\lfloor 8}$

$\Rightarrow \quad (n-8)(n-9)(n-10)(n-11) = 12 \times 11 \times 10 \times 9$

$\Rightarrow \quad n = 20$

Hence $\quad c(22, n) = c(22, 20)$

$= \dfrac{\lfloor 22}{\lfloor 20 \lfloor 22-20} = \dfrac{21 \times 22}{\lfloor 2} = 231$

26. $\because A = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$

Now $A^2 = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix}$

$A^3 = \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix} \cdot \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = \begin{bmatrix} \omega^3 & 0 \\ 0 & \omega^3 \end{bmatrix}$

Hence $A^{100} = \begin{bmatrix} \omega^{100} & 0 \\ 0 & \omega^{100} \end{bmatrix}$

$$= \begin{bmatrix} \omega^{3\times 33+1} & 0 \\ 0 & \omega^{3\times 33+1} \end{bmatrix}$$

$$= \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} \qquad (\because \omega^3 = 1)$$

$$= A$$

27. Let $z = \dfrac{1+2i}{1-(1-i)^2}$

then $|z| = \dfrac{|1+2i|}{|1-(1-i)^2|} = \dfrac{|1+2i|}{|1-1-i^2+2i|} = \dfrac{|1+2i|}{|1+2i|} = 1$

28. We have $(-\sqrt{-1})^{4n+3} + (i^{41} + i^{-257})^9$, $n \in N$

$$= (-i)^{4n+3} + \left(i + \frac{1}{i}\right)^9$$

$$= (-1)^{4n+1} i^{4n+3} + (i-i)^9$$

$$= (-1)(-i) + 0$$

$$= i$$

29. Here $x = (1101)_2$

$$= 1\times 2^3 + 1\times 2^2 + 0\times 2^1 + 1\times 2^0$$

$$= 8 + 4 + 0 + 1 = 13$$

and $y = (110)_2 = 1\times 2^2 + 1\times 2^1 + 0\times 2^0$

$$= 4 + 2 + 0 = 6$$

Hence $x^2 - y^2 = (13)^2 - 6^2 = 169 - 36 = 133$

Now

2	133	
2	66	1
2	33	0
2	16	1
2	8	0
2	4	0
2	2	0
	1	0

$\Rightarrow 133 = (10000101)_2$

30. $\because (10x\,010)_2 - (11y)_2 = (10z\,11)_2$

$\Rightarrow$ $(2^5\times 1 + 2^4\times 0 + 2^3\times x + 1\times 2^2$
$+ 1\times 2^1 + 0\times 2^0) - (2^3\times 1 + 2^2\times 1 + y\times 2^1 + 1\times 2^0)$
$= 2^4\times 1 + 0\times 2^3 + 2^2\times z + 2^1\times 1 + 2^0$

$\Rightarrow$ $(34 + 8x) - (13 + 2y) = 19 + 4z$

$\Rightarrow$ $2 = -8x + 2y + 4z \Rightarrow 1 = -4x + y + 2z$

$\Rightarrow$ $x = 0, y = 1, z = 0$

31. Given equation is $(x-p)(x-q) = r^2$

$\Rightarrow$ $x^2 - (p+q)x + pq - r^2 = 0$

Hence $= \sqrt{(p+q)^2 - 4(pq - r^2)}$

$= \sqrt{(p-q)^2 + 4r^2} \geq 0$

$\Rightarrow$ Roots of the given equation are always real.

32. Here $(\sin x + \operatorname{cosec} x)^2 + (\cos x + \sec x)^2$
$= k + \tan^2 x + \cot^2 x$

$\Rightarrow$ $\sin^2 x + \operatorname{cosec}^2 x + 2\sin x \operatorname{cosec} x$
$+ \cos^2 x + \sec^2 x + 2\cos x \sec x$
$= k + \tan^2 x + \cot^2 x$

$\Rightarrow$ $1 + \operatorname{cosec}^2 x - \cot^2 x + \sec^2 x - \tan^2 x + 4 = k$

$\Rightarrow$ $1 + 1 + 1 + 4 = k$

$\Rightarrow$ $k = 7$

33. Here, the order of given matrices are
$[X]_{(a+b)\times(a+2)}$ and $[Y]_{(b+1)(a+3)}$

As $[XY]$ and $[YX]$ exist.

Hence $a + 2 = b + 1$ and $a + 3 = a + b$

$\Rightarrow$ $a = 2, b = 3$

34. Let n teams participated in the championship.

Hence $^nC_2 = 153.$

$\Rightarrow$ $\dfrac{n(n-1)}{2} = 153$

$\therefore$ $n(n-1) = 306$

$\Rightarrow$ $n = 18$

35. The given equation is
$x - 2(x-1)^{-1} = 1 - 2(x-1)^{-1}$

$\Rightarrow$ $x - \dfrac{2}{x-1} = 1 - \dfrac{2}{x-1}$

$\Rightarrow$ $x = 1$

But the given equation is not satisfied at $x = 1$.

Hence no roots exist.

36. Here $0.0011 = 0\times\dfrac{1}{2} + 0\times\left(\dfrac{1}{2}\right)^2 + 1\times\left(\dfrac{1}{2}\right)^3 + 1\times\left(\dfrac{1}{2}\right)^4$

$$= 0 + 0 + \frac{1}{8} + \frac{1}{16} = \frac{3}{16}$$

Clearly option (c) is correct.

37. We have $y = \tan^{-1} x - x$

$\Rightarrow$ $\dfrac{dy}{dx} = \dfrac{1}{1+x^2} - 1 = -\dfrac{x^2}{1+x^2}$

$\Rightarrow$ $\dfrac{dy}{dx} < 0, \forall x \in R$

Hence the given function is always decreasing.

38. Given $n(A) = 115$, $n(B) = 326$, $n(A - B) = 47$

Now $n(A \cap B) = n(A) - n(A - B)$
$= 115 - 47 = 68$

Now $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow$ $= 115 + 326 - 68$
$= 373$

39. Here $x\,dy = y\,dx$

or $\dfrac{dy}{y} = \dfrac{dx}{x}$

On integrating,

$= \lim_{x \to \infty} \left(1 + \frac{3}{x}\right)^{\frac{x}{3}(-9)} = e^{-9}$

48. Here $f(x) = \begin{cases} -x^2, & x < 0 \\ x, & x \geq 0 \end{cases}$

Now LHL $= \lim_{x \to 0^-} f(x) = -\lim_{x \to 0} x^2 = 0$

and RHL $= \lim_{x \to 0^+} f(x) = \lim_{x \to 0} x = 0$

and $f(0) = 0$

clearly LHL = RHL = $f(0)$

Hence $f(x)$ is continuous at every $x \in R$.

49. Given $\sqrt{1-x^2} + \sqrt{1-y^2} = a$

$\Rightarrow \frac{1}{2\sqrt{1-x^2}}(-2x) + \frac{1(-2y)}{2\sqrt{1-y^2}} \frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{x}{y}\sqrt{\frac{1-y^2}{1-x^2}}$

50. Here $x = \log t$, $y = t^2 - 1$

Now using $x = \log t$

$\Rightarrow \quad 2x = \log t^2$

$\Rightarrow \quad 2x = \log(1+y)$

$\Rightarrow \quad e^{2x} = 1 + y \quad \ldots(i)$

Now differentiating (i), we get

$e^{2x}\, 2 = \frac{dy}{dx}$

Now again differentiating, we get

$4e^{2x} = \frac{d^2y}{dx^2}$

at $\quad t = 1, x = 0$

Hence $\frac{d^2y}{dx^2} = 4\, e^{2 \times 0} = 4e^0 = 4$

51. Considering option (d)

$F(x) = -x$ for all $x \in R$

For every values of x, we get different value of x.

Hence it is injective.

52. Let $t_1 = \log x^5$ and $t_2 = \log_5 x$

$\Rightarrow \quad t_1 = \frac{\log_e 5}{\log_e x} \quad \ldots(i)$

and $\quad t_2 = \frac{\log_e x}{\log_e 5} \quad \ldots(ii)$

Differentiating (i) w.r.t. x we get

$\frac{dt_1}{dx} = -\frac{\log_e 5}{x(\log_e x)^2}$

Now differentiating (ii) w.r.t. x, we get

$\frac{dt_2}{dx} = \frac{1}{x \log_e 5}$

Hence $\frac{dt_1}{dt_2} = -\frac{-\log_e 5}{x(\log_e x)^2} \times x \log_e 5$

$= -\left(\frac{\log_e 5}{\log_e x}\right)^2$

$= -(\log_x 5)^2$

$= -(\log_5 x)^{-2}$

53. Here $v = s + 1$

then $\quad \frac{ds}{dt} = s + 1$

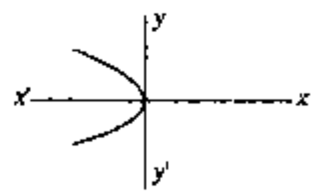
or $\quad \frac{ds}{s+1} = dt$

Now integrating

$\log(s+1) = t$

At $s = 9$ m $\Rightarrow t = \log(9+1) = \log 10\, s$

54. The given curve is $y^2 = -4ax$



which is a parabola.

Clearly given curve lies in the second and third quadrants.

55. The given equation is

$x^2 + y^2 + 4x - 4y + 4 = 0$

$(x+2)^2 + (y-2)^2 = 2^2$

Here centre of given circle is

(2, 2) and radius = 2

Hence it touches both the axes.

56. Here $\cos 60 = \frac{1 \times 1 + 1 \times -1 + 1 \times n}{\sqrt{1^2+1^2+1^2}.\sqrt{1^2+(-1)^2+n^2}}$

$\Rightarrow \quad \frac{1}{2} = \frac{1-1+n}{\sqrt{3}.\sqrt{2+n^2}}$

$\Rightarrow \quad 2n = \sqrt{3}.\sqrt{2+n^2}$

$\Rightarrow \quad 4n^2 = 3(2+n^2)$

$\Rightarrow \quad n^2 = 6$

$\Rightarrow \quad n = \pm\sqrt{6}$

57. Given line is

$ax\cos\phi + by\sin\phi = ab$

Now at point $(b^2 - a^2, 0)$

$D_1 = \frac{a\sqrt{b^2-a^2}\cos\phi - ab}{\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}} \quad \ldots(i)$

and at point $(-\sqrt{b^2-a^2}, 0)$

$$D_2 = \frac{-a\sqrt{b^2-a^2}\cos\phi - ab}{\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}} \quad ...(ii)$$

Multiplying (i) and (ii) we get

$$D_1D_2 = -\frac{a^2(b^2-a^2)\cos^2\phi - a^2b^2}{a^2\cos^2\phi + b^2\sin^2\phi}$$

$$= \frac{-a^2(-b^2\sin^2\phi - a^2\cos^2\phi}{a^2\cos\phi + b^2\sin^2\phi}$$

$$= a^2$$

58. Given, the middle point of the segment of the straight line joining the points (p, q) and $(q, -p)$ is $\left(\frac{r}{2}, \frac{s}{2}\right)$

$\Rightarrow \quad \frac{p+q}{2} = \frac{r}{2} \quad \Rightarrow p+q = r$

and $\quad \frac{q-p}{2} = \frac{s}{2} \quad \Rightarrow q-p = s$

$\therefore$ Length of the segment $= \sqrt{(p-q)^2 + (q+p)^2}$

$= \sqrt{(s^2+r^2)}$

59. The equation of plane passing through $(1, -2, 4)$ and normal $(2, 1, 2)$ is

$2(x-1) + 1(y+2) + 2(z-4) = 0$

$\Rightarrow \quad 2x + y + 2z - 8 = 0$

Hence required distance $= \frac{2.3 + 1.2 + 2.3 - 8}{\sqrt{2^2+1^2+2^2}}$

$= \frac{6+2+6-8}{3} = 2$

60. Here, equation of the plane passing through $(1, -3, 1)$ and normal $(1, -3, 1)$ is

$1(x-1) - 3(y+3) + 1(z-1) = 0$

$\Rightarrow \quad x - 3y + z - 11 = 0$

$\Rightarrow \quad \frac{x}{11} - \frac{3y}{11} + \frac{z}{11} = 1$

Clearly the intercept cut on the x axis by the plane is 11.

61. We know

$l^2 + m^2 + n^2 = 1$

$\Rightarrow \quad \cos^2\alpha + \cos^2\alpha + \cos^2\theta = 1 \quad ...(i)$

Now given $\sin^2\theta = 2\sin^2\alpha$

$\Rightarrow \quad 1 - \cos^2\theta = 2(1-\cos^2\alpha)$

$\Rightarrow \quad \cos^2\theta = 2\cos^2\alpha - 1$

Putting this value in eq. (1), we get,

$\cos^2\alpha + \cos^2\alpha + 2\cos^2\alpha - 1 = 1$

$\Rightarrow \quad 4\cos^2\alpha = 2$

$\Rightarrow \quad \cos\alpha = \pm\frac{1}{2} \Rightarrow \alpha = \frac{\pi}{4}, \frac{3\pi}{4}$

62. Let the locus of the required point is (x_1, y_1)

$\sqrt{[x_1-(m+n)]^2 + [y_1-(n-m)]^2}$

$= \sqrt{[x_1-(m-n)]^2 + [y_1-(n+m)]^2}$

$\Rightarrow \quad x_1^2 + (m+n)^2 - 2x_1(m+n) + y_1^2 + (n-m)^2 - 2y_1(n-m)$

$= x_1^2 + (m-n)^2 - 2x_1(m-n) + y_1^2 + (n+m)^2 - 2y_1(n+m)$

$\Rightarrow \quad 2x_1(m-n-m-n) + 2y_1(n+m-n+m) = 0$

$\Rightarrow \quad -4x_1n + 4y_1m = 0$

$\Rightarrow \quad nx_1 = my_1$

Hence the locus of the given point is

$nx = my.$

63. Here centre of sphere is $(6, -1, 2)$

then radius of sphere $= \frac{2\times 6 - 1(-1) + 2(2) - 2}{\sqrt{2^2+1^2+2^2}} = \frac{15}{3} = 5$

Hence equation of sphere is

$(x-6)^2 + (y+1)^2 + (z-2)^2 = 5^2$

$\Rightarrow \quad x^2 + y^2 + z^2 - 12x + 2y - 4z + 16 = 0$

(on solving)

64. Here the intersection of given planes is

$x - y + 2z - 1 + \lambda(x + y - z - 3) = 0$

$\Rightarrow \quad x(1+\lambda) + y(\lambda-1) + z(2-\lambda) - 3\lambda - 1 = 0$

$\therefore$ Direction ratios of normal to the above plane is

$(1+\lambda, \lambda-1, 2-\lambda)$

Now considering option (a) $(-1, 3, 2)$

$\Rightarrow \quad -1(1+\lambda) + 3(\lambda-1) + 2(2-\lambda) = 0$

$\Rightarrow \quad -1 - \lambda + 3\lambda - 3 + 4 - 2\lambda = 0$

$\Rightarrow \quad 0 = 0$

$\Rightarrow$ option (a) is correct.

65. Given equation is $\frac{x^2}{169} + \frac{y^2}{25} = 1$

$\therefore \quad e_1 = \sqrt{1 - \frac{25}{169}} = \frac{12}{13}$

and $\quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\therefore \quad e_2 = \sqrt{1 - \frac{b^2}{a^2}}$

But given $e_1 = e_2$

$\Rightarrow \quad \frac{12}{13} = \sqrt{1 - \frac{b^2}{a^2}}$

$\Rightarrow \quad \frac{b^2}{a^2} = 1 - \frac{144}{169} = \frac{169-144}{169} = \frac{25}{169}$

$\Rightarrow \quad \frac{a}{b} = \frac{13}{5}$

66. Here, the projection or $\bar{b}$ on $\bar{a} = \frac{\bar{a}.\bar{b}}{|\bar{a}|} = \frac{\bar{a}.\bar{b}}{1} = \bar{a}.\bar{b}$

67. The given vector is $4\hat{i} - 3\hat{j} + \hat{k}$

Now considering option (a)

$$\pm\left(\frac{3\hat{i} + 4\hat{j}}{5}\right).(4\hat{i} - 3\hat{i} + \hat{k}) = \frac{12-12}{5} = \frac{0}{5} = 0$$

$\therefore$ option (a) is true.

68. Here given vector is $\vec{r} = a\hat{i} + b\hat{j}$

Now considering option (d)

$$(b\hat{i} - a\hat{j}).(a\hat{i} + b\hat{j}) = b.a - ab = 0$$

Hence option (d) is correct.

69. Here $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$
and $\vec{b} = m\vec{a}$

$= m(2\hat{i} - 3\hat{i} + 4\hat{k})$

As $\vec{b}$ is $\vec{a}$ unit vector

$\Rightarrow$ $|2\hat{i} - 3\hat{i} + 4\hat{k}| = \sqrt{4+9+16} = \sqrt{29}$

Hence $m = \frac{1}{\sqrt{29}}$

70. Given the vectors $\lambda\vec{a} + \vec{b}$ and $\vec{a} - \lambda\vec{b}$ are perpendicular to each other.

$\Rightarrow$ $(\lambda\vec{a} + \vec{b}).(\vec{a} - \lambda\vec{b}) = 0$

$\Rightarrow$ $\lambda|\vec{a}|^2 - \lambda|\vec{b}|^2 + (1-\lambda^2)\,\vec{a}.\vec{b} = 0$

$\therefore$ $(1-\lambda^2)|\vec{a}||\vec{b}|\cos 60 = 0$ $[\because |\vec{a}| = |\vec{b}|$ and $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta]$

$\Rightarrow$ $1 - \lambda^2 = 0$

$\Rightarrow$ $\lambda = \pm 1$

71. We know that
$|\vec{a} + \vec{b}|^2 + |\vec{a} - \vec{b}|^2 = 2[|\vec{a}|^2 + |\vec{b}|^2]$

$\Rightarrow$ $|\vec{a} + \vec{b}|^2 + 7^2 = 2[3^2 + 4^2]$

$\Rightarrow$ $|\vec{a} + \vec{b}|^2 = 50 - 49$

$\Rightarrow$ $|\vec{a} + \vec{b}| = 1$

72. Let $\vec{D}_1 = 3\hat{i} + 6\hat{j} - 2\hat{k}$
and $\vec{D}_2 = 4\hat{i} - \hat{j} + 3\hat{k}$

Now $\vec{D}_1\vec{D}_2 = 3.4 + 6(-1) + (-2).(3)$

$= 0$

and $|\vec{D}_1| = \sqrt{3^2 + 6^2 + (-2)^2} = 7$

$|\vec{D}_2| = \sqrt{4^2 + (-1)^2 + (3)^2} = \sqrt{26}$

Clearly $|\vec{D}_1| \neq |\vec{D}_2|$

Hence given quadrilateral at must be a rhombus.

73. We have $p = \sin(99)^\circ.\cos(991^\circ)$
$= \sin(1080 - 91) - \cos(1080 - 89)$
$= -\sin 91^\circ \cos 89^\circ$
$= -\cos 1^\circ \cos 89^\circ$

Clearly p is finite and negative.

74. Here $\frac{1 - 3\tan^2 A}{3\tan A - \tan^3 A} = \frac{1}{\tan 3A}$

$= \frac{1}{\tan\frac{3.41\pi}{12}}$ $\left(\because A = \frac{41\pi}{12}\right)$

$= \frac{1}{\tan\frac{41\pi}{4}} = \frac{1}{\tan\left(10\pi + \frac{\pi}{4}\right)} = \frac{1}{\tan\frac{\pi}{4}} = 1$

75. Let $z = 2\omega^2 + 3i$
$= -1 + (3 - \sqrt{3})i$

Hence $\bar{z}$ = conjugate of z

$= -1 - (3 - \sqrt{3})i$

$= -1 + i\sqrt{3} - i3$

$= 2\omega - 3i$

76. I. We have $(\sec\theta + \tan\theta)^{-1} = (\sec 1200 + \tan 1200)^{-1}$

$= [\sec(6 \times \pi + 120) + \tan(6\pi + 120)]^{-1}$

$= (\sec 120 + \tan 120)^{-1}$

$= (-2 - \sqrt{3})^{-1}$

$= -\frac{1}{(2 + \sqrt{3})}$ = negative value

and II. $\text{cosec}\,\theta - \cot\theta = \text{cosec}\,120^\circ - \cot 120^\circ$

$= \text{cosec}(6\pi + 120) - \cot(6\pi + 120)$

$= \text{cosec}\,120 - \cot 120$

$= \sec 30 + \tan 30$

$= \frac{2}{\sqrt{3}} + \sqrt{3}$ = positive value

Hence (d) is the correct answer.

77. We have

$\cot\theta = 2\cos\theta, \frac{\pi}{2} \angle \theta \angle \pi$

$\Rightarrow$ $\cot\theta - 2\cos\theta = 0$

$\Rightarrow$ $\cot\theta(1 - 2\sin\theta) = 0$

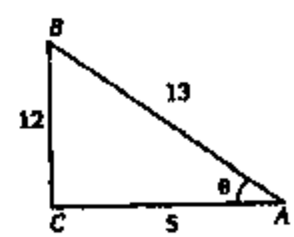
Here $\cot\theta \neq 0$, where $\pi/2 \angle \theta \angle \pi$

then $1 - 2\sin\theta = 0$

$\Rightarrow$ $\sin\theta = \frac{1}{2}$

$\Rightarrow$ $\sin\theta = \sin\frac{5\pi}{6} \Rightarrow \theta = \frac{5\pi}{6}$

78. Here $\cot\theta = 5/12$



Hence $\cos\theta = \frac{-5}{13}$, ($\because \theta$ lies in III Quadrant)

$\sin\theta = -\frac{12}{13}$

Now $2\sin\theta + 3\cos\theta = 2\left(\frac{-12}{13}\right) + 3\left(\frac{-5}{13}\right)$

$= \frac{-24}{13} - \frac{15}{13}$

$= -3$

$\therefore$(d) is correct answer.

79. We have

$\cos\frac{\pi}{9} + \cos\frac{\pi}{3} + \cos\frac{5\pi}{9}.\cos\frac{7\pi}{9} = \cos 20 + \cos 60 + \cos 100 + \cos 140$

$= \cos 20 + \frac{1}{2} + 2\cos(120)\cos 20$

$= \cos 20 + \frac{1}{2} - 2\sin 30 \cos 20$

$= \frac{1}{2}$

80. We know

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$ (Let)

$\Rightarrow \sin A = ak, \sin C = ck$

Now $\cos B = \frac{\sin A}{2\sin C}$

$\Rightarrow \frac{a^2 + c^2 - b^2}{2ac} = \frac{ak}{2ck}$

$\Rightarrow a^2 + c^2 - b^2 = a^2$

$\Rightarrow b^2 = c^2$

$\Rightarrow b = c$

Hence the given triangle is isosceles.

81. According to the question

$\frac{28\times1400}{100} + \frac{35\times1400}{100} + \frac{12\times1400}{100} + \frac{8\times1400}{100} + 105 + \text{Transport} = 1400$

$\Rightarrow 392 + 490 + 168 + 112 + 105 + \text{Transport} = 1400$

$\Rightarrow 1267 + \text{Transport} = 1400$

$\Rightarrow$ Transport = Rs.133 crores

82. Here C carried $= \frac{51}{75} \times 100 = 68$ verange marks out of 100.

Hence average percentage marks

$= 35\times74 + 35\times70 \frac{+30\times68}{35+35+30}$

$= \frac{2590 + 2450 + 2040}{100}$

$= 70.80$

83. Here education upto primary school = 27%

and education upto middle school = 18.6%

and education upto graduate $= \frac{660}{15000}\times100 = 4.4$

Hence total = 50%

and total literate people in town = 100 – 35.4 = 64.6%

Let people with education upto pre university = x

then people with education upto high school = $2x$

Hence $2x + x = (64.6 - 50)\%$ of 15000

$\Rightarrow 3x = \frac{14.6}{100}\times15000$

$\Rightarrow 3x = 2190$

$\Rightarrow x = 730$

Hence total people with education upto high school

$= 2x = 2\times730 = 1460$

84. Here required probability $= \frac{^{25}C_3}{^{26}C_3} = \frac{25\times24\times23}{26\times25\times24}$

$= \frac{23}{26}$

85. Here (b) is the correct answer.

86. $\because$ lines of regression passes through $(\bar{X}, \bar{Y})$

Hence, $3\bar{X} + \bar{Y} - 12 = 0$

and $\bar{X} + 2\bar{Y} - 14 = 0$

On solving above equation we get

$\bar{X} = 2$ and $\bar{Y} = 6$

87. We know coefficient of variance $= \frac{\sqrt{SD}}{\text{mean}}$

$\therefore$coefficient of variance $A = \frac{\sqrt{12}}{61} = \frac{3.46}{61} = 0.057$

and coefficient of variance $B = \frac{\sqrt{25}}{90} = \frac{5}{90} = 0.055$

coefficient of variance $C = \frac{\sqrt{36}}{80} = \frac{6}{80} = 0.075$

coefficient of variance $D = \frac{\sqrt{16}}{120} = \frac{4}{120} = 0.033$

Clearly for product D the demand is consistent.

88. Taking 3, 4, 5, 6, 7 as five integens

$\therefore$ mean $= \frac{3+4+5+6+7}{5} = \frac{25}{5} = 5$

Now

$SD = \sqrt{\frac{(5-3)^2 + (5-4)^2 + (5-5)^2 + (5-6)^2 + (5-7)^2}{5}}$

$= \sqrt{\frac{10}{5}} = \sqrt{2}$

89. Required number of elementary events $= {}^7C_1 \times {}^6C_1$

$$= \frac{\lfloor 7}{\lfloor 1 \lfloor 6} \times \frac{\lfloor 6}{\lfloor 1 \lfloor 5}$$

$$= \frac{7 \times 6 \lfloor 5}{\lfloor 5} = 42$$

90. Given $P(A) = 0.8, P(B) = 0.7$

Now $P(A \cup B) \le 1$

$\Rightarrow \quad P(A) + P(B) - P(A \cap B) \le 1$

$\Rightarrow \quad 0.8 + 0.7 - P(A \cap B) \le 1$

$\Rightarrow \quad P(A \cap B) \le 1.5 - 1$

$\Rightarrow \quad P(A \cap B) \ge 0.5$

91. Here required probability $= \frac{n(E)}{n(S)} = \frac{6}{6} = 1$

92. Since $x, 2x + 2, 3x + 3$ are in G.P.

$\therefore \quad (2x + 2)^2 = x.(3x + 3)$

$\Rightarrow \quad x^2 + 5x + 4 = 0$

$\Rightarrow \quad x = -1, -4$

Hence first term $a = x$

and second term $= ar = 2(x + 1)$

$\Rightarrow \quad r = \frac{2(x+1)}{x}$

Now fourth term $T_4 = ar^3$

$$= x\left(\frac{2(x+1)}{x}\right)^3$$

(Putting the values of a and r)

Now substituting $x = 4$, we get

$$T_4 = -4\left[\frac{2(-4+1)}{-4}\right]^3$$

$$= \frac{-4 \times 8 \times -27}{-64}$$

$$= -\frac{27}{2}$$

93. Here (c) is the correct option.

94. Here both the statements are correct.

95. Given, $P(A) = \frac{1}{3}, P(B) = \frac{3}{4}, P(A \cup B) = \frac{11}{12}$

Now $P(A \cap B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{3} + \frac{3}{4} - \frac{11}{12} = \frac{1}{6}$$

Hence $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{6}}{\frac{1}{3}} = \frac{1}{2}$

96. Given equation is

$(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$

$\Rightarrow \quad 3x^2 - 2(b + a + c)x + ab + bc + ca = 0$

$\therefore \quad D = \sqrt{[-2(b + a + c)]^2 - 4.3.(ab + bc + ca)}$

$$= 2\sqrt{a^2 + b^2 + c^2 - ab - bc - ca}$$

$$= 2\sqrt{\tfrac{1}{2}\{(a-b)^2 + (b-c)^2 + (c-a)^2\}}$$

$$\ge 0$$

97. $(\log_x x.|(\log_3 2x)(\log_{2x} y) = \log_x x^2$

$\Rightarrow \quad 1.(\log_3 2x)(\log_{2x} y) = 2$

$\Rightarrow \quad \log_3 y = 2$

$\Rightarrow \quad y = 3^2 = 9$

98. Here $\log_k x : \log_5 k = 3$

$\Rightarrow \quad \log_5 x = 3$

$\Rightarrow \quad x = 5^3 = 125$

99. Given series is

$$20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots$$

or $\quad 20, \frac{77}{4}, \frac{37}{2}, \frac{71}{4}, \ldots$

which is an AP series

Here $a = 20, d = -\frac{3}{4}$

Now $T_n = a + (n-1)d$

$$= 20 + (n-1)\frac{-3}{4}$$

$$= \frac{83}{4} - \frac{3}{4}n$$

Now for first negative term $T_n < 0$

$\Rightarrow \quad \frac{83}{4} - \frac{3}{4}n < 0$

$\Rightarrow \quad 83 < 3n$

$\Rightarrow \quad n > \frac{83}{3} \Rightarrow \quad n > 28$

Hence n should be 28.

$\therefore$ 28th term is first negative term.

100. Given $\sin^{-1} x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$

$\Rightarrow \quad \sin^{-1} x + \cos^{-1}\frac{1}{\sqrt{5}} = \frac{\pi}{2}$

Now comparing by $\sin^{-1}\alpha + \cos^{-1}\alpha = \frac{\pi}{2}$

$\Rightarrow \quad x = \frac{1}{\sqrt{5}}$

101. Let α is common root.

$\therefore \quad \alpha^2 + m\alpha + 1 = 0$

and $\quad \alpha^2 + \alpha + m = 0$

solving $\quad \frac{\alpha^2}{m^2 - 1} = \frac{\alpha}{1 - m} = \frac{1}{1 - m}$

$\Rightarrow \quad \frac{\alpha}{1 - m} = \frac{1}{1 - m} \Rightarrow \alpha = 1$

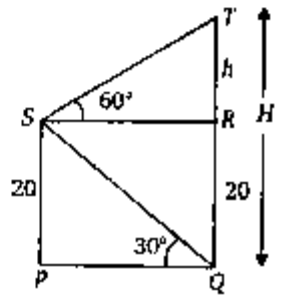
Again $\quad \frac{\alpha^2}{m^2 - 1} = \frac{1}{1 - m} \Rightarrow \frac{1}{m^2 - 1} = \frac{1}{1 - m}$

$\Rightarrow \quad 1 - m = m^2 - 1$

$\Rightarrow \quad m^2 + m - 2 = 0$

$\Rightarrow \quad m = 1 \text{ and } -2$

102. Let H is the height of the tower.



Now in PQS,

$$\tan 30° = \frac{20}{PQ}$$

$\Rightarrow \quad PQ = 20\sqrt{3}$ m

and in ΔSTR

$$\tan 60 = \frac{h}{SR} = \frac{h}{PQ} = \frac{h}{20\sqrt{3}}$$

$\Rightarrow \quad \sqrt{3} = \frac{h}{20\sqrt{3}} \Rightarrow h = 60$ m

$\therefore$ Height of tower $= H = TR + RQ$

$= 60 + 20$

$= 80$ m

103. We have $\sqrt{3}$ cosec $20° - \sec 20°$

$$\Rightarrow \quad 2\left[\frac{\sqrt{3}}{2}\text{cosec } 20 - \frac{\sec 20}{2}\right]$$

$$= 2\,\frac{\frac{\sqrt{3}}{2}\cos 20 - \frac{1}{2}\sin 20}{\sin 20 \cos 20}$$

$$= \frac{2\sin(60-20)}{\frac{1}{2}\sin 40} = \frac{4\sin 40}{\sin 40} = 4$$

104. $\therefore$ Here $\tan 15 = \tan(45-30) = \dfrac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}} = 2 - \sqrt{3}$

B_1 $\tan 75 = \tan(30+45)$

$$= \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}} = 2 + \sqrt{3}$$

and $\tan 105 = \tan(45+60)$

$$= \frac{1+\sqrt{3}}{1-\sqrt{3}} = -2 - \sqrt{3}$$

Hence (b) $A - 4B - 2C - 1$ is correct.

105. Here $a + b = 3(1+\sqrt{3})$

and $a - b = 3(1-\sqrt{3})$

on solving we get

$a = 3, b = 3\sqrt{3}$

We know sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Hence $\dfrac{3}{\sin 30} = \dfrac{3\sqrt{3}}{\sin B}$

$\Rightarrow \sin B = \dfrac{3\sqrt{3}}{3}\cdot\dfrac{1}{2} = \dfrac{\sqrt{3}}{2} = \sin 60$

$\Rightarrow \quad B = 60°$

106. Here $N_a = \{ax \mid x \in N\}$

$\therefore \quad N_{12} = \{12, 24, 36, 48 \ldots\}$

$N_8 = \{8, 16, 24, \ldots.\}$

Hence $N_{12} \cap N_8 = \{24, 48 \ldots\} = N_{24}$

107. Given $X = \{(4^n - 3n - 1) \mid n \in N\}$

$\Rightarrow \quad X = \{0, 9, 54 \ldots.\}$

and $\quad Y = \{9(n-1) \mid n \in N\}$

$= \{0, 9, 18, 27, 36, 54 \ldots\}$

Hence $X \cup Y = \{0, 9, 18, 27, 36 \ldots$

$= Y$

108. Here $(d) n^2$ is the correct option.

109. Here $z + z^{-1} = 1$

or $\quad z^2 - z + 1 = 0$

$\Rightarrow \quad z = -\omega, -\omega^2$

Now when $z = -\omega$

$\Rightarrow \quad z^{99} + z^{-99} = (-\omega)^{99} + (-\omega)^{-99}$

$= -1 - 1 = -2$

and when $z = -\omega^2$

$\Rightarrow \quad z^{99} + z^{-99} = (-\omega^2)^{99} + (-\omega^2)^{-99}$

$= -1 - 1 = -2$

$\therefore$ (d) is the correct answer.

110. Given in an AP the mth term is $\dfrac{1}{n}$

$\therefore \quad T_m = a + (m-1)d$

$\Rightarrow \quad \dfrac{1}{n} = a + (m-1)d \quad \ldots$(i)

and n th term is $\dfrac{1}{m}$

$\Rightarrow \quad \dfrac{1}{m} = a + (n-1)d \quad \ldots$(ii)

Solving (i) and (ii), we get

$a = d = \dfrac{1}{mn}$

Hence $T_{mn} = \frac{1}{mn} + (mn-1)\frac{1}{mn} = 1$

Hence (d) is the correct answer.

111. Here (b) is the correct option.

112. I. Here coefficient of the middle term in the expansion of $(1+x)^8$ is $^8C_4$

and coefficient of the middle term in the expansion of $\left(x+\frac{1}{x}\right)^8$ is $^8C_4$

Hence statement I is correct.

II. Here coefficient of the middle term in the expansion of $(1+x)^8$ is $^8C_4$

and coefficient of the fifth term in the expansion of $(1+x)$ is $^7C_4$

clearly $^8C_4 > {^7C_4}$

Hence II is not correct;

113. Given equation is

$$xy = ae^x + be^{-x}$$

On differentiating

$\Rightarrow \quad y + x\frac{dy}{dx} = ae^x - be^{-x}$

Again on differentiating, we get

$$\frac{dy}{dx} + \frac{dy}{dx} + x\frac{d^2y}{dx^2} = ae^x + be^{-x}$$

$\Rightarrow \quad x\frac{d^2y}{dx^2} + \frac{2dy}{dx} = xy$

which is of order 2 and degree 1.

114. The given equation is

$$y = (1+x^{1/4})(1+x^{1/2})(1-x^{1/4})$$

$$= (1+x^{1/2})(1^2-(x^{1/4})^2)$$

$$= (1+x^{1/2})(1-x^{1/2})$$

$$= [(1)^2-(x^{1/2})^2) = 1-x$$

Now $\frac{dy}{dx} = 0-1 = -1$

115. Here $v = x^2 \log\frac{1}{x}$

$$= -x^2 \log x$$

Now $\frac{dv}{dx} = -2x\log x - \frac{x^2}{x} = -2x\log x - x$

and $\frac{d^2v}{dx^2} = -\frac{2x}{x} - 2\log x - 1$

$$= -3 - 2\log x$$

For maxima or minima put $\frac{dv}{dx} = 0$

$\Rightarrow \quad -2x\log x - x = 0$

$\Rightarrow \quad x = e^{-1/2}$

at $x = e^{-1/2}$ $\frac{d^2v}{dx^2} = -3-2\left(-\frac{1}{2}\right) = -2$ (maxima)

116. Given equation is $\quad y = 4x - x^2 - 3 \quad$ ...(i)

Now put $x = 1$ in (i) we get

$y = 0$

and $x = 3$, we get

$y = 0$

Hence required area $= \int_1^3 y\, dx$

$$= \int_1^3 (4x - x^2 - 3)\, dx$$

$$= \left(2x^2 - \frac{x^3}{3} - 3x\right)_1^3$$

$$= \frac{4}{3} \text{ sq. unit} \quad \text{(on solving)}$$

117. We have $f'(x) = 6 - 4\sin 2x$

$\Rightarrow \quad \int f'(x)\,dx = \int[6-4\sin 2x)]\,dx$

$\Rightarrow \quad f(x) = 6x + 2\cos 2x + C$

Put $x = 0$ we get

$f(0) = 6.0 + 2.1 + c$

$\Rightarrow \quad 3 = 2 + c \quad \Rightarrow \quad c = 1$

Hence $f(x) = 6x + 2\cos 2x + 1$

118. Here $(gof)(x) = g\ f(x)$

$= g(e^x)$

$= \log e^x$

$= x$

$\therefore \quad (gof)'(x) = 1$

119. We have $f'(x) = g'(x)$

On integrating

$\Rightarrow \quad f(x) = g(x) + c$

$\Rightarrow \quad f(x) = x^3 - 4x + 6 + c$

$\because \quad f(1) = 2$

Hence $2 = 1 - 4 + 6 + c$

$\Rightarrow \quad c = -1$

$f(x) = x^3 - 4x + 6 - 1$

$= x^3 - 4x + 5$

120. Here $f(x) = \begin{cases} \frac{|x|}{x} & x \ne 0 \\ 2 & x = 0 \end{cases}$

$$= \begin{cases} 1 & x > 0 \\ 2 & x = 0 \\ -1 & x < 0 \end{cases}$$

Hence range of f is $\{-1, 1, 2\}$.