TEST SERIES-1

1.	(d)	2.	(c)	3.	(b)	4.	(c)	5.	(c)	6.	(c)	7.	(d)	8.	(a)	9.	(d)	10.	(d)
11.	(a)	12.	(b)	13.	(a)	14.	(c)	15.	(a)	16.	(c)	17.	(a)	18.	(c)	19.	(c)	20.	(d)
21.	(d)	22.	(d)	23.	(a)	24.	(d)	25.	(c)	26.	(d)	27.	(b)	28.	(d)	29.	(a)	30.	(b)
31.	(a)	32.	(c)	33.	(c)	34.	(d)	35.	(b)	36.	(a)	37.	(b)	38.	(b)	39.	(b)	40.	(d)
41.	(d)	42.	(b)	43.	(c)	44.	(d)	45.	(b)	46.	(b)	47.	(d)	48.	(d)	49.	(c)	50.	(b)
51.	(d)	52.	(d)	53.	(b)	54.	(b)	55.	(c)	56.	(c)	57.	(a)	58.	(a)	59.	(a)	60.	(b)
61.	(a)	62.	(b)	63.	(b)	64.	(b)	65.	(a)	66.	(a)	67.	(c)	68.	(a)	69.	(a)	70.	(b)
71.	(b)	72.	(d)	73.	(c)	74.	(c)	75.	(b)	76.	(c)	77.	(c)	78.	(a)	79.	(d)	80.	(d)
81.	(a)	82.	(b)	83.	(b)	84.	(b)	85.	(a)	86.	(b)	87.	(b)	88.	(c)	89.	(d)	90.	(a)
91.	(d)	92.	(b)	93.	(c)	94.	(c)	95.	(b)	96.	(a)	97.	(a)	98.	(b)	99.	(a)	100.	(b)
101.	(a)	102.	(c)	103.	(b)	104.	(b)	105.	(b)	106.	(d)	107.	(c)	108.	(b)	109.	(d)	110.	(a)
111.	(a)	112.	(d)	113.	(b)	114.	(a)	115.	(b)	116.	(b)	117.	(b)	118.	(a)	119.	(b)	120.	(b)

1. Let, α and γ be the roots of $A x^2 - 4x + 1 = 0$

$\therefore \quad \alpha + \gamma = \frac{4}{A}$ and $\alpha\gamma = \frac{1}{A}$

and β and δ be the roots of $Bx^2 - 6x + 1 = 0$

$\therefore \quad \beta + \delta = \frac{6}{B}$ and $\beta\delta = \frac{1}{B}$

Also, α, β, γ and δ are in HP.

$\therefore \quad \frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ and $\frac{1}{\delta}$ are in AP.

$$\Rightarrow \quad \frac{1}{\beta} - \frac{1}{\alpha} = \frac{1}{\delta} - \frac{1}{\gamma}$$

$$\Rightarrow \quad \frac{1}{\beta} - \frac{1}{\delta} = \frac{1}{\alpha} - \frac{1}{\gamma}$$

$$\Rightarrow \quad \frac{\delta - \beta}{\beta\delta} = \frac{\gamma - \alpha}{\alpha\gamma}$$

$$\Rightarrow \quad \frac{\sqrt{(\delta + \beta)^2 - 4\beta\delta}}{\beta\delta} = \frac{\sqrt{(\gamma + \alpha)^2 - 4\alpha\gamma}}{\alpha\gamma}$$

$$\Rightarrow \frac{\sqrt{\left(\frac{6}{B}\right)^2 - 4 \cdot \frac{1}{B}}}{\frac{1}{B}} = \frac{\sqrt{\left(\frac{4}{A}\right)^2 - 4 \cdot \frac{1}{A}}}{\frac{1}{A}}$$

$$\Rightarrow \quad \sqrt{\frac{36}{B^2} - \frac{4}{B}} \times \frac{B}{1} = \sqrt{\frac{16}{A^2} - \frac{4}{A}} \times \frac{A}{1}$$

$$\Rightarrow \quad \sqrt{36 - 4B} = \sqrt{16 - 4A}$$

$$\Rightarrow \quad 36 - 4B = 16 - 4A$$

$$\Rightarrow \quad 4A + 4B = 36 - 16 = 20$$

$$\Rightarrow \quad A + B = 5$$

So, $\quad A = -3$ and $B = 8$ (possible)

2. Given that,

$$kx + y + z = k - 1$$
$$x + ky + z = k - 1$$
$$x + y + kz = k - 1$$

$$\therefore \quad A = \begin{bmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{bmatrix}, B = \begin{bmatrix} k-1 \\ k-1 \\ k-1 \end{bmatrix} \text{ and } x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Now, $|A| = \begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix}$

$$= k(k^2 - 1) - 1(k - 1) + 1(1 - k)$$
$$= k^3 - k - k + 1 + 1 - k$$
$$= k^3 - 3k + 2$$

The given system of equations has no solution, if $|A| = 0$

$$\Rightarrow \quad k^3 - 3k + 2 = 0$$

$$\Rightarrow \quad (k - 1)^2 (k + 2) = 0$$

$$\Rightarrow \quad k = 1 \text{ or } k = -2$$

3. We know that largest side has greatest angle opposite it.

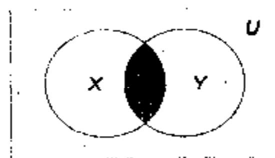
Here, $a = 6$ cm, $b = 10$ cm and $c = 14$ cm

$$\therefore \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{(6)^2 + (10)^2 - (14)^2}{2 \times 6 \times 10}$$

$$= \frac{36 + 100 - 196}{2 \times 6 \times 10} = -\frac{1}{2} = \cos 120°$$

$$\Rightarrow \quad \angle C = 120°$$

4. From the figure,



It is clear that, $(X - Y)' = X' \cup Y$

5. For finding the area of a triangle ABC, angles A, B and side c are required.

6. Given that, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$$\therefore \quad AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$$

and $\quad BA = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & 2a \\ 3b & 4b \end{bmatrix}$

If $\quad AB = BA$

$$\Rightarrow \quad \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix} = \begin{bmatrix} a & 2a \\ 3b & 4b \end{bmatrix} \Rightarrow a = b$$

So, it is clear that there exist infinitely many B's such that $AB = BA$.

7. Given that, $M = \begin{bmatrix} 3 & 4 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & k \end{bmatrix}$

Now, $|M| = \begin{vmatrix} 3 & 4 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & k \end{vmatrix} = k(3 - 8) = -5k$

If $k \neq 0$, then inverse of M exist. So, statement A implies B as well as B implies A.

8. Given that, $\quad 2^x + 3^y = 17 \quad \ldots$(i)

and $\quad 2^{x-2} - 3^{y+1} = 5$

$$\Rightarrow \quad 4 \cdot 2^x - 3 \cdot 3^y = 5 \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii),

$$2^x = 8 \text{ and } 3^y = 9$$

$$\Rightarrow \quad 2^x = 2^3 \text{ and } 3^y = 3^2$$

$\Rightarrow \quad x = 3$ and $y = 2$

9. Given that, $P(32,6) = kC(32,6)$

$$\Rightarrow \quad {}^{32}P_6 = k\,{}^{32}C_6$$

$$\Rightarrow \quad \frac{32!}{(32-6)!} = k \cdot \frac{32!}{6!(32-6)!}$$

$$\Rightarrow \quad k = 6! = 720$$

10. Given that, $\dfrac{\sqrt{3}+i}{1+\sqrt{3}i} = \dfrac{(\sqrt{3}+i)(1-\sqrt{3}i)}{(1+\sqrt{3}i)(1-\sqrt{3}i)}$

$$= \frac{\sqrt{3}-3i+i+\sqrt{3}}{1+3}$$

$$= \frac{2\sqrt{3}-2i}{4} = \frac{\sqrt{3}-i}{2}$$

11. **From option (a),**

$$(0.1101)_2 = 1\times 2^{-1} + 1\times 2^{-2} + 1\times 2^{-4}$$

$$= \frac{1}{2}+\frac{1}{4}+\frac{1}{16} = \frac{13}{16}$$

$$= (0.8125)_{10}$$

$$\therefore \quad (0.8125)_{10} = (0.1101)_2$$

12. Given that, $\begin{vmatrix} y & x & y+z \\ z & y & x+y \\ x & z & z+x \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} x+y+z & x+y+z & 2(x+y+z) \\ z & y & x+y \\ x & z & z+x \end{vmatrix} = 0$$

$$\Rightarrow \quad (x+y+z)\begin{vmatrix} 1 & 1 & 2 \\ z & y & x+y \\ x & z & z+x \end{vmatrix} = 0$$

$$\Rightarrow \quad (x+y+z)\begin{vmatrix} 1 & 0 & 0 \\ z & z-y & x+y-2z \\ x & z-x & z-x \end{vmatrix} = 0$$

$$\Rightarrow \quad (x+y+z)\begin{vmatrix} z-y & x+y-2z \\ z-x & z-x \end{vmatrix} = 0$$

$$\Rightarrow (x+y+z)(z-x)(z-y-x-y+2z) = 0$$

$$\Rightarrow \quad x+y = -z$$

or $\quad z = x$

13. Suppose that $\Delta = \begin{vmatrix} k & b+c & b^2+c^2 \\ k & c+a & c^2+a^2 \\ k & a+b & a^2+b^2 \end{vmatrix}$

$$= k\begin{vmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ b^2+c^2 & c^2+a^2 & a^2+b^2 \end{vmatrix}$$

$$= k\begin{vmatrix} 1 & 0 & 0 \\ b+c & a+b & a-c \\ b^2+c^2 & a^2-b^2 & a^2-c^2 \end{vmatrix}$$

$$= k\begin{vmatrix} 1 & 0 & 0 \\ b+c & a-b & a-c \\ b^2+c^2 & (a-b)(a+b) & (a-c)(a+c) \end{vmatrix}$$

$$= k(a-b)(a-c)\begin{vmatrix} 1 & 1 \\ a+b & a+c \end{vmatrix}$$

$$= k(a-b)(a-c)(a+c-a-b)$$

$$= k(a-b)(b-c)(c-a)$$

But $\Delta = (a-b)(b-c)(c-a)$

$\therefore \quad k = 1$

14. The total number of proper subsets of a given finite set with n elements

$$= 2^n - 1$$

15. Given that, $(x+a)$ is a factor of x^2+px+q and x^2+lx+m

$$\therefore \quad a^2 - ap + q = 0 \quad \text{...(i)}$$

and $\quad a^2 - la + m = 0 \quad$ ...(ii)

From Eqs. (i) and (ii), we get

$$-ap + q + la - m = 0$$

$$\Rightarrow \quad (l-p)a = m - q$$

$$\therefore \quad a = \frac{m-q}{l-p}\ (l \neq p)$$

16. We know that

$$[(A\cup B)\cap C]' = A'\cap B' \cup C'$$

17. $\tan(-1575^\circ) = -\tan(4\times 360^\circ + 135^\circ)$

$$= -\tan 135^\circ$$

$$= -\tan(90^\circ + 45^\circ)$$

$$= \cot 45^\circ = 1$$

18. Given that, $\operatorname{cosec}^2\theta = 3\sqrt{3}\cot\theta - 5$

$$\Rightarrow 1 + \cot^2\theta - 3\sqrt{3}\cot\theta + 5 = 0$$

$$(\because \operatorname{cosec}^2\theta = 1 + \cot^2\theta)$$

$$\Rightarrow \quad \cot^2\theta - 3\sqrt{3}\cot\theta + 6 = 0$$

This equation is satisfied by $\theta = \dfrac{\pi}{6}$

$$\therefore \quad \theta = \frac{\pi}{6}$$

19. $\because \cos 2\phi - 1 = \dfrac{1-\tan^2\phi}{1+\tan^2\phi} - 1$

$$= -\frac{2\tan^2\phi}{1+\tan^2\phi}$$

$$= \frac{-(\tan^2\theta - 1)}{1 + \dfrac{\tan^2\theta - 1}{2}}$$

$$= \frac{1-\tan^2\theta}{1+\tan^2\theta}\times 2 = \cos 2\theta \cdot 2$$

Thus, $\cos 2\theta = \dfrac{\cos 2\phi - 1}{2}$

$\therefore$ Option (c) is correct.

20. $\because \quad A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$$\therefore \quad |A| = \begin{vmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{vmatrix} = -1(-1) = 1 \neq 0$$

$\therefore \quad A^{-1}$ exists

Now, $\quad A^2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad A^2 = I$$

21. $\sin^{-1}\{2x(1-x^2)\} = 2\sin^{-1}x$ is true

$$\forall x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$$

22. $1 - \sin 10^\circ \sin 50^\circ \sin 70^\circ$

$$= 1 - \frac{1}{2}[2\sin 70^\circ \sin 10^\circ \sin 50^\circ]$$

$$= 1 - \frac{1}{2}[(\cos 60^\circ - \cos 80^\circ)\sin 50^\circ]$$

$$= 1 - \frac{1}{2}\left[\frac{1}{2}\sin 50^\circ - \frac{1}{2}\cos 80^\circ \sin 50^\circ\right]$$

$$= 1 - \frac{1}{4}[\sin 50^\circ - \sin 130^\circ + \sin 30^\circ]$$

$$= 1 - \frac{1}{8} = \frac{7}{8}$$

23. Given that, $\sin\theta = \dfrac{5}{13}$ and $\sin\phi = \dfrac{99}{101}$

$$\therefore \cos\{\pi - (\theta+\phi)\}$$

$$= -\cos(\theta+\phi)$$

$$= -\{\cos\theta\cos\phi - \sin\theta\sin\phi\}$$

$$= -\left\{\sqrt{1-\left(\frac{5}{13}\right)^2}\sqrt{1-\left(\frac{99}{101}\right)^2} - \frac{5}{13}\times\frac{99}{101}\right\}$$

$$= -\left\{\sqrt{1-\frac{25}{169}}\cdot\sqrt{1-\frac{9801}{10201}} - \frac{495}{1313}\right\}$$

$$= \left\{\sqrt{\frac{144}{169}}\cdot\sqrt{\frac{400}{1021}} - \frac{495}{1313}\right\}$$

$$= -\left\{\frac{12}{13}\times\frac{20}{101} - \frac{495}{1313}\right\}$$

$$= -\left\{\frac{240}{1313} - \frac{495}{1313}\right\} = \frac{255}{1313}$$

24. $\because 1000^\circ = 2\times 360^\circ + 280^\circ$

Thus, it is clear that the revolving line will be situated in fourth quadrant.

25. **We know that**

1 radian = 57° 17′ 44.8″ ≈ 57° (Approx.)

26. Given that, $\cot(x+y) = \dfrac{1}{\sqrt{3}} = \cot 60^\circ$

$$\Rightarrow \quad x + y = 60^\circ \quad \text{...(i)}$$

and $\cot(x-y)=\sqrt{3}=\cot 30^\circ$

$\Rightarrow \quad x-y=30^\circ \quad \ldots(ii)$

From Eqs. (i) and (ii), we get

$x=45^\circ$ and $y=15^\circ$

27. Given that, $\sin A=\dfrac{1}{\sqrt{5}}$ and $\cos B=\dfrac{3}{\sqrt{10}}$

$\therefore \sin(A+B)=\sin A\cos B+\cos A\sin B$

$$=\frac{1}{\sqrt{5}}\times\frac{3}{\sqrt{10}}+\sqrt{1-\frac{1}{5}}\times\sqrt{1-\frac{9}{10}}$$

$$=\frac{3}{\sqrt{50}}+\frac{2}{\sqrt{5}}\times\frac{1}{\sqrt{10}}$$

$$=\frac{3}{\sqrt{50}}+\frac{2}{\sqrt{50}}$$

$$=\frac{3+2}{\sqrt{50}}=\frac{1}{\sqrt{2}}$$

$$=\sin\frac{\pi}{4}$$

$\therefore \quad A+B=\dfrac{\pi}{4}$

28. Given that,

$$\sin^{-1}\left(\frac{2a}{1+a^2}\right)-\cos^{-1}\left(\frac{1-b^2}{1+b^2}\right)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$\Rightarrow \quad 2\tan^{-1}a-2\tan^{-1}b=2\tan^{-1}x$

$\Rightarrow \quad \tan^{-1}a-\tan^{-1}b=\tan^{-1}x$

$\Rightarrow \quad \tan^{-1}\left(\dfrac{a-b}{1+ab}\right)=\tan^{-1}x$

$\therefore \quad x=\dfrac{a-b}{1+ab}$

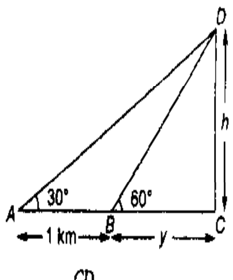
29. Given that,

$x=\sin\theta\cos\theta$ and $y=\sin\theta+\cos\theta$

$\therefore y^2-2x=(\sin\theta+\cos\theta)^2-2(\sin\theta\cos\theta)$

$=\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-2\sin\theta\cos\theta=1$

30. In ΔBDC,



$\tan 60^\circ=\dfrac{CD}{BC}$

$\Rightarrow \quad \sqrt{3}=\dfrac{h}{y}$

$\therefore \quad h=\sqrt{3}y \quad \ldots(i)$

and now in ΔADC,

$\tan 30^\circ=\dfrac{CD}{AC}$

$\Rightarrow \quad \dfrac{1}{\sqrt{3}}=\dfrac{h}{1+y}$

$\Rightarrow \quad 1+y=h\sqrt{3}$

$\Rightarrow \quad 1+y=3y \quad$ [from Eq. (i)]

$\therefore \quad y=\dfrac{1}{2}$

$\therefore \quad h=\dfrac{\sqrt{3}}{2}$ km $\quad$ [from Eq. (i)]

31. Put points A, B, C and D as coordinates $(1, 3, 4), (-1, 6, 10), (-7, 4, 7)$ and $(-5, 1, 1)$ respectively.

$\therefore \quad AB=\sqrt{(-1-1)^2+(6-3)^2+(10-4)^2}$

$=\sqrt{(-2)^2+(3)^2+(6)^2}$

$=\sqrt{4+9+36}=\sqrt{49}=7$

$BC=\sqrt{(-7+1)^2+(4-6)^2+(7-10)^2}$

$=\sqrt{(-6)^2+(-2)^2+(-3)^2}$

$=\sqrt{36+4+9}=\sqrt{49}=7$

$CD=\sqrt{(-5+7)^2+(1-4)^2+(1-7)^2}$

$=\sqrt{(2)^2+(-3)^2+(-6)^2}$

$=\sqrt{4+9+36}=\sqrt{49}=7$

$DA=\sqrt{(1+5)^2+(3-1)^2+(4-1)^2}$

$=\sqrt{(6)^2+(2)^2+(3)^2}$

$=\sqrt{36+4+9}=\sqrt{49}=7$

$AC=\sqrt{(-7-1)^2+(4-3)^2+(7-4)^2}$

$=\sqrt{(-8)^2+(1)^2+(3)^2}$

$=\sqrt{64+1+9}=\sqrt{74}$

and $BD=\sqrt{(-5+1)^2+(1-6)^2+(1-10)^2}$

$=\sqrt{(-4)^2+(-5)^2+(-9)^2}$

$=\sqrt{16+25+81}$

$=\sqrt{122}$

Hence, $AB=BC=CD=DA$

But $\quad BD\neq AC$

$\therefore$ Points A, B, C and D are the vertices of a rhombus.

32. We know that the number of planes passing through three non-collinear points is 1.

33. The angle between the lines $x+y=0, y=0$ and $20x=15y=12z$ is $\sin^{-1}(1/5)$.

34. According to question, Latusrectum of an ellipse $=\dfrac{2b^2}{a}$

and minor axis $=2b$

$\therefore \quad b=\dfrac{2b^2}{a}$

$\Rightarrow \quad a=2b$

Also, $e=\sqrt{1-\dfrac{b^2}{a^2}}=\sqrt{1-\dfrac{b^2}{(2b)^2}}$

$=\sqrt{1-\dfrac{b^2}{4b^2}}=\sqrt{1-\dfrac{1}{4}}=\sqrt{\dfrac{3}{4}}$

$=\dfrac{\sqrt{3}}{2}$

35. The given equation

$x^2+y^2+z^2+2Ux+2Uy+2Wz+d=0$

represents a real sphere, if

$u^2+v^2+w^2>d$

36. Clearly option (a) is correct.

37. We know that

$|\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2=(|\vec{a}|^2\times|\vec{b}|^2)$

$\Rightarrow (8)^2+|\vec{a}\cdot\vec{b}|^2=[(2)^2\times(5)^2]$

$\therefore \quad 64+|\vec{a}\cdot\vec{b}|^2=(4\times 25)$

$\Rightarrow \quad |\vec{a}\cdot\vec{b}|^2=36=100-64$

$\Rightarrow \quad \vec{a}\cdot\vec{b}=6$

38. Given that, $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$

$\Rightarrow \quad |\vec{a}+\vec{b}|^2=|\vec{a}-\vec{b}|^2$

$\Rightarrow \quad |\vec{a}|^2+|\vec{b}|^2+2|\vec{a}|\cdot|\vec{b}|$

$=|\vec{a}|^2+|\vec{b}|^2-2|\vec{a}|\cdot|\vec{b}|$

$\Rightarrow \quad 4|\vec{a}|\cdot|\vec{b}|=0$

$\Rightarrow \vec{a}$ is perpendicular to $\vec{b}$.

39. Given that, $\vec{a}=\hat{i}-2\hat{j}+5\hat{k}$

and $\quad \vec{b}=2\hat{i}+\hat{j}-3\hat{k}$

$\therefore \quad \vec{b}-\vec{a}=2\hat{i}+\hat{j}-3\hat{k}-\hat{i}+2\hat{j}-5\hat{k}$

$=\hat{i}+3\hat{j}-8\hat{k}$

and $\quad (3\vec{a}+\vec{b})=(3\hat{i}-6\hat{j}+15\hat{k})+(2\hat{i}+\hat{j}-3\hat{k})$

$=5\hat{i}-5\hat{j}+12\hat{k}$

$\therefore (\vec{b}-\vec{a})\cdot(3\vec{a}+\vec{b})=(\hat{i}+3\hat{j}-8\hat{k})\cdot(5\hat{i}-5\hat{j}+12\hat{k})$

$=5-15-96=5-111$

$=-106$

40. We know that Points A, B and C are collinear, if

$(\vec{a}\times\vec{b})+(\vec{b}\times\vec{c})+(\vec{c}\times\vec{a})=\vec{0}$

41. Given that, $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}-\hat{k}$

$\therefore \quad \vec{a}\times(\vec{b}+\vec{c})+\vec{b}\times(\vec{c}+\vec{a})+\vec{c}\times(\vec{a}+\vec{b})$

$=\vec{a}\times\vec{b}+\vec{a}\times\vec{c}+\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{a}+\vec{c}\times\vec{b}$

$$= \vec{a}\times\vec{b} - \vec{c}\times\vec{a} + \vec{b}\times\vec{c} - \vec{a}\times\vec{b} + \vec{c}\times\vec{a} - \vec{b}\times\vec{c}$$

$$= \vec{0}$$

42. Required event $= A \cap B \cap \bar{C}$

43. The sales are most consistent During month 3

44. We know that, conditional probability is calculated by Baye's theorem.

45. Given that, $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}, P\left(\frac{A}{B}\right) = \frac{1}{6}$

$\therefore \quad P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)}$

$\Rightarrow \quad \frac{1}{6} = \frac{P(A\cap B)}{\frac{1}{4}}$

$\Rightarrow \quad P(A\cap B) = \frac{1}{24}$

$\therefore \quad P\left(\frac{B}{A}\right) = \frac{P(A\cap B)}{P(A)}$

$= \frac{1/24}{1/3} = \frac{1}{8}$

46. If A and B are mutually exclusive and exhaustive events, then

$P(A\cap B) = 0, P(A\cup B) = 1$

we know that

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\Rightarrow \quad 1 = P(A) + 3P(A) - 0 \quad [\because P(B) = 3P(A)]$

$\Rightarrow \quad 1 = 4P(A)$

$\Rightarrow \quad P(A) = \frac{1}{4}$

$\therefore \quad P(B) = \frac{3}{4}$

Hence, $P(\bar{B}) = 1 - P(B)$

$= 1 - \frac{3}{4} = \frac{1}{4}$

47. $\because n(S) = 36$

E = Sum of the faces equals or exceeds

$= \{(5,5), (4,6), (6,4), (5,6), (6,5), (6,6)\}$

$\therefore \; n(E) = 6$

We know that $P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

48. Given that, $\quad np = 4$ and $npq = \frac{4}{3}$

$\therefore \quad 4q = \frac{4}{3}$

$\Rightarrow \quad q = \frac{1}{3}$

$\therefore \quad p = 1 - \frac{1}{3} = \frac{2}{3}$

$\therefore \quad n = \frac{4}{p} = \frac{4\times 3}{2} = 6$

Now, $P(X \geq 5) = {}^6C_5 (p)^5 (q)^1 + {}^6C_6\, p^6 q^0$

$= {}^6C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^1 + {}^6C_6 \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^0$

$= \frac{6\times 32}{3^6} + \frac{64}{3^6} = \frac{256}{3^6} = \frac{2^8}{3^6}$

49. Given that, $H = 21.6$ and $a = 27$

We know that

$H = \frac{2ab}{a+b}$

$\Rightarrow \quad 21.6 = \frac{2\times 27\times b}{27+b}$

$\Rightarrow \quad 583.2 + 21.6b = 54b$

$\Rightarrow \quad 583.2 = 54b - 21.6b = 32.4b$

$\Rightarrow \quad b = \frac{583.2}{32.4} = 18$

50. Average scores of A

$= \frac{71+56+55+75+54+49}{6} = \frac{360}{6} = 60$

and

$$SD = \sqrt{\frac{(60-71)^2 + (60-56)^2 + (60-55)^2 + (60-75)^2 + (60-54)^2 + (60-49)^2}{6}}$$

$= \sqrt{\frac{121+16+25+225+36+121}{6}}$

$= \sqrt{\frac{544}{6}} = 9.52$

Also, average of marks of B

$= \frac{55+74+83+54+38+52}{6}$

$= \frac{356}{6} = 59.33$

Hence, the average scores of A and B are not same but A is consistent.

51. Here, $n = 50, x = 3550, n_1 = 30, x_1 = 4050$ and $n_2 = 20$

We know that, $\quad nx = n_1x_1 + n_2x_2$

$\Rightarrow \quad 50\times 3550 = 30\times 4050 + 20\,x_2$

$\Rightarrow \quad 177500 - 121500 = 20x_2$

$\therefore \quad x_2 = 2800$

Hence, average salary of women = Rs 2800

52. $\bar{x} = \frac{7+9+11+13+15}{5} = \frac{55}{5} = 11$

Now,

$$SD = \sqrt{\frac{(7-11)^2 + (9-11)^2 + (11-11)^2 + (13-11)^2 + (15-11)^2}{5}}$$

$= \sqrt{\frac{16+4+0+4+16}{5}}$

$= \sqrt{8} = 2.8$ (approx)

53. Here, $n(S) = 52$ and $n(E) = 4$

We know that $\quad P(E) = \frac{n(E)}{n(S)} = \frac{4}{52}$

$= \frac{1}{13}$

54. Given that monthly salary = Rs 15000

and sector angle of transport expenses = 15°

$\therefore$ Monthly expenditureon transport

$= \frac{15^\circ}{360^\circ}\times 15000 = $ Rs 625

55. Given that, $\sum_{i=1}^{n} (x_i - 2) = 110$

$\therefore \quad x_1 + x_2 + \ldots + x_n - 2n = 110$

$\Rightarrow \quad x_1 + x_2 + \ldots + x_n = 2n + 110 \quad \ldots(i)$

and $\quad \sum_{i=1}^{n} (x_i - 5) = 20$

$\Rightarrow \quad x_1 + x_2 + \ldots + x_n - 5n = 20$

$\Rightarrow \quad x_1 + x_2 + \ldots + x_n = 5n + 20 \quad \ldots(ii)$

From Eqs. (i) and (ii), we get

$5n + 20 = 2n + 110$

$\Rightarrow \quad 3n = 90 \;\; \therefore\; n = 30$

Now, mean $= \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{5n+20}{n}$ [From Eq (ii)]

$= \frac{5\times 30 + 20}{30} = \frac{170}{30} = \frac{17}{3}$

56. Given that, $f(x) = x|x|$

If $\quad f(x_1) = f(x_2)$

$\Rightarrow \quad x_1|x_1| = x_2|x_2|$

$\Rightarrow \quad x_1 = x_2$

$\therefore\; f(x)$ is one-one.

Also, range of $f(x)$ = co-domain of $f(x)$

$\therefore\; f(x)$ is onto.

Hence, $f(x)$ is both one-one and onto.

57. Given that $\; f(x) = \frac{x}{1+|x|}$

$$= \begin{cases} \frac{x}{1-x}, & x < 0 \\ \frac{x}{1+x}, & x \geq 0 \end{cases}$$

$\therefore$ LHD $= f'(0^-) = \lim_{h\to 0} \frac{f(0-h) - f(0)}{-h}$

$= \lim_{h\to 0} \frac{\frac{-h}{1+h} - 0}{-h}$

$= \lim_{h\to 0} \frac{1}{1+h} = 1$

and $\quad$ RHD $= f'(0^+)$

$= \lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$

$= \lim_{h\to 0} \frac{\frac{h}{1+h} - 0}{h}$

$= \lim_{h\to 0} \frac{1}{1+h} = 1$

$\because$ LHD = RHD

$\therefore$ $f(x)$ is differentiable at $x=0$

Hence, $f(x)$ is differentiable in $(-\infty,\infty)$.

58. $\lim\limits_{\delta x\to 0}\dfrac{\delta y}{\delta x}=\left(\dfrac{dy}{dx}\right)_{\text{at } x=0}$

$$=\left(\frac{da}{dx}x^n\right)_{\text{at } x=0}$$

$$=(an\,x^{n-1})_{\text{at } x=0}=0$$

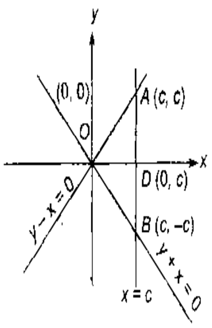
59. Given that, $\dfrac{dy}{dx}=e^{x-y}(e^{y-x}-e^y)$

$=e^{-y}\cdot e^y(e^x\cdot e^{-x}-e^x)$

$\Rightarrow$ $\int 1dy=\int(1-e^x)\,dx$

$\Rightarrow$ $y=x-e^x+c$

60. Required area of the triangle



$=2$ area (ΔAOD)

$=2\times\dfrac{1}{2}\times OD\times AD$

$=c\times c$

$=c^2$

61. Given that, $A\subseteq X$ and $B\subseteq X$

$\therefore\{(A\cap(X-B))\}\cup B$

$=(A\cap B')\cup B$

$=A\cup B$

62. Let sets A and B have m and n elements respectively.

$\therefore$ $2^m-2^n=56$

$\Rightarrow$ $2^n(2^{m-n}-1)=8\times 7$

$\Rightarrow$ $n=3$ and $m-n=3$

$\Rightarrow$ $m=6$ and $n=3$

63. We know that, 9! = 362880

which is not divisible by 990.

Now, 11! = 39916800

which is divisible by 990.

Thus, the smallest natural number is 11

64. We know that

$$A\times(B-C)=(A\times B)-(A\times C)$$

65. Here, $n(E)=75$, $n(M)=60$ and $n(E\cap M)=45$

We know that $n(E\cup M)=n(E)+n(M)-n(E\cap M)$

$=75+60-45=90$

Thus, required number students

$=90-45=45$

66. Here, $R=\{3,6,9,12,15,\ldots,99\}$

and $S=\{5,10,15,\ldots,95\}$

Now, $(R\times S)\cap(S\times R)$

$=(R\cap S)\times(S\cap R)$

$=\{15,30,45,60,75,90\}\times\{15,30,45,60,75,90\}$

$\therefore$ Number of elements in $(R\times S)\cap(S\times R)=6\times 6=36$

67. Here, $(a,a),(b,b),(c,c)\in R$

$\therefore$ R is reflexive relation.

But $(a,b)\in R$ and $(b,a)\notin R$.

$\therefore$ R is not symmetric relation.

Also, $(a,b),(b,c)\in R$

$\Rightarrow$ $(a,c)\in R$

$\therefore R$ is not transitive relations.

68. Given that, $\log_{10}(x+1)+\log_{10}5=3$

$\Rightarrow$ $\log_{10}5(x+1)=3$

$\Rightarrow$ $(x+1)=\dfrac{1000}{5}=200$

$\therefore$ $x=200-1=199$

69. $2\log_8 2-\dfrac{\log_3 9}{3}$

$=\dfrac{2}{3}\log_2 2-2\dfrac{\log_3 3}{3}$

$=\dfrac{2}{3}-\dfrac{2}{3}=0$

70. Given that, $(b-c)x^2+(c-a)x+(a-b)=0$

$\Rightarrow$ $(b-c)x^2-(b-c-b+a)x+(a-b)=0$

$\Rightarrow$ $(b-c)x(x-1)-(a-b)(x-1)=0$

$\Rightarrow$ $\{(b-c)x-(a-b)\}\{x-1\}=0$

$\Rightarrow$ $x=\dfrac{a-b}{b-c}$ and $x=1$

71. Given that, $16\left(\dfrac{a-x}{a+x}\right)^3=\dfrac{a+x}{a-x}$

$\Rightarrow$ $\left(\dfrac{a-x}{a+x}\right)^4=\left(\dfrac{1}{2}\right)^4$

$\Rightarrow$ $\dfrac{a-x}{a+x}=\dfrac{1}{2}$

$\Rightarrow$ $2a-2x=a+x$

$\therefore$ $a=3x$

and $x=\dfrac{a}{3}$

72. Given that, α and β be the roots of

$2x^2-2(1+n^2)x+(1+n^2+n^4)=0$

$\therefore$ $\alpha+\beta=(n^2+1)$

and $\alpha\beta=\dfrac{1+n^2+n^4}{2}$

Now, $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=(n^2+1)^2-(1+n^2+n^4)$

$=n^4+1+2n^2-1-n^2-n^4=n^2$

73. If, r and s are the roots of $Ax^2+Bx+C=0$, then

$r+s=-\dfrac{B}{A}$ and $rs=\dfrac{C}{A}$

Now, r^2 and s^2 to be the roots of $x^2+px+q=0$

Then $r^2+s^2=-p$ and $r^2s^2=q$

$\Rightarrow$ $(r+s)^2-2rs=-p$

$\Rightarrow$ $\dfrac{B^2}{A^2}-\dfrac{2C}{A}=-p$

$\Rightarrow$ $\dfrac{B^2-2AC}{A^2}=-p$

$\therefore$ $p=\dfrac{2AC-B^2}{A^2}$

74. Then that, $P(5,r)=P(6,r-1)$

${}^5P_r={}^6P_{r-1}$

$\Rightarrow$ $\dfrac{5!}{(5-r)!}=\dfrac{6!}{(6-r+1)!}$

$\Rightarrow$ $(7-r)(6-r)=6$

$\Rightarrow$ $42-13r+r^2=6$

$\Rightarrow$ $r^2-13r+36=0$

$\Rightarrow$ $r^2-9r-4r+36=0$

$\Rightarrow$ $(r-9)(r-4)=0$

$\therefore$ $r=4$ $(\because r\neq 9)$

75. Possibilities of words formed from the letters of word "JOKE" are

JOKE, KOJE, KEJO, JEKO, EJOK, EKOJ, OKEJ, OJEK

Hence, required number of words = 8

76. According to questions,

$a+ar=8$

$\Rightarrow$ $a(1+r)=8$ ...(i)

and $a+ar+ar^2+ar^3=80$

$\Rightarrow$ $a(1+r)+ar^2(1+r)=80$

$\Rightarrow$ $a(1+r)(1+r^2)=80$

$\Rightarrow$ $1+r^2=\dfrac{80}{8}=10\Rightarrow r^2=9$

$\therefore$ $r=3$ $(\because r>0)$

From Eq. (i), $a(1+3)=8$

$\therefore$ $a=2$

then, $T_6=ar^5=2(3)^5=2\times 243=486$

77. $(101.101)_2=1\times 2^2+0\times 2^1+1\times 2^0+1\times 2^{-1}$
$+0\times 2^{-2}+1\times 2^{-3}$

$=4+1+\dfrac{1}{2}+\dfrac{1}{8}$

$=\dfrac{40+4+1}{8}=\dfrac{45}{8}$

$=(5.625)_{10}$

78. According to qeustions, $\log_2 x, \log_3 x, \log_x 16$ are in GP.

$\therefore \quad (\log_3 x)^2 = \log_2 x \cdot \log_x 16$

$\Rightarrow \quad (\log_3 x)^2 = \log_2 16$

$\Rightarrow \quad (\log_3 x)^2 = 4\log_2 2$

$\Rightarrow \quad \log_3 x = 2$

$\therefore \quad x = 3^2 = 9$

79. Put T_{r+1} be the term independent of x in $\left(\frac{3x^{-2}}{2} - \frac{1}{3x}\right)^9$.

$$\therefore \quad T_{r+1} = {}^9C_r\left(\frac{3x^{-2}}{2}\right)^{9-r}\left(-\frac{1}{3x}\right)^r$$

$$= (-1)^r\, {}^9C_r\left(\frac{3}{2}\right)^{9-r} \cdot \frac{1}{3^r} x^{-18+2r-r}$$

To be term independent from x,

$-18 + r = 0$

$\Rightarrow \quad r = 18$

which is not possible.

Hence, no such term exists in the expansion.

80. We know that First five terms of a geometric progression are a, ar, ar^2, ar^3, ar^4.

$$\therefore \quad \text{Mean} = \frac{a + ar + ar^2 + ar^3 + ar^4}{5}$$

$$= \frac{a(r^5 - 1)}{5(r-1)}$$

81. Given that, $2x = 3 + 5i$, $x = \frac{3+5i}{2}$

$$\Rightarrow \quad x^3 = \frac{27 + 125i^3 + 225i^2 + 135i}{8}$$

$$= \frac{27 - 125i - 225 + 135i}{8}$$

$$= \frac{-198 + 10i}{8}$$

$$= \frac{-99 + 5i}{4}$$

and $$x^2 = \frac{9 + 25i^2 + 30i}{4}$$

$$= \frac{9 - 25 + 30i}{4} = \frac{-8 + 15i}{2}$$

Now, $2x^3 + 2x^2 - 7x + 72$

$$= \left(\frac{-99 + 5i}{2}\right) + (-8 + 15i) - \frac{7(3 + 5i)}{2} + 72$$

$$= -\frac{99}{2} + \frac{5i}{2} - 8 + 15i - \frac{21}{2} - \frac{35}{2}i + 72$$

$$= \left(-\frac{99}{2} - 8 - \frac{21}{2} + 72\right) + \left(\frac{5}{2} + 15 - \frac{35}{2}\right)i$$

$$= \frac{-99 - 16 - 21 + 144}{2}$$

$$= \frac{-136 + 144}{2} = \frac{8}{2} = 4$$

82. Here, $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

Then, $A\,(\text{adj}\,A) = I_2|A|$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

83. Put $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$\therefore \quad |A| = -1$

and $\quad \text{adj}\,A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

Hence, $A^{-1} = \frac{1}{|A|}\text{adj}\,A$

$$= -\frac{1}{1}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

84. $\because (AB)' = B'A'$

$\therefore$ AB is not symmetric.

and $\quad (A^2 + B^2)' = (A')^2 + (B')^2 = A^2 + B^2$

$\therefore$ $A^2 + B^2$ is symmetric

Hence, statement (2) is correct.

85. (A) $\frac{dy}{dx} = 3x^2 - 2x - 1$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(1,1)} = 3 - 2 - 1 = 0$$

The equation of tangent is

$y - 1 = 0(x - 1) \Rightarrow y = 1$

ie, parallel to x-axis.

Hence, Both A and R true and R is the correct explanatio of A.

86. (A) We know that

Work done $= \vec{\mathbf{F}} \cdot \vec{\mathbf{a}} = |\vec{\mathbf{F}}| \cdot |\vec{\mathbf{d}}| \cos\theta$

Since, $\quad \theta = 90°$

$$= \vec{\mathbf{F}} \cdot \vec{\mathbf{d}} = |\vec{\mathbf{F}}| \cdot |\vec{\mathbf{d}}| \cos 90° = 0$$

(R) $\quad \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = 0$

$\Rightarrow \vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are perpendicular.

Hence, Both A and R are true but R is not corre explanation of A.

87. (A) Required probability $= \frac{4}{52} + \frac{4}{52} = \frac{1}{13} + \frac{1}{13} = \frac{2}{13}$

(R) $= P(E_1 + E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

Hence, Both A and R are true but R is not the corre explanation A.

88. (A) $\left[\frac{-1 + \sqrt{-3}}{2}\right]^{29} + \left[\frac{-1 - \sqrt{-3}}{2}\right]^{29}$

From Eqs. (i) and (ii), we get

$$2I = \int_0^{\frac{\pi}{2}} \log(\tan x \cot x)\, dx$$

$= 0$

$\Rightarrow \quad I = 0$

98. Put $\quad I = \int \tan^2 x \sec^4 x\, dx$

Again let $\tan x = t$ and $\sec^2 x\, dx = dt$

$$\therefore \quad I = \int t^2(1 + t^2)\, dt = \int (t^2 + t^4)\, dt$$

$$= \frac{t^5}{5} + \frac{t^3}{3} + c$$

$$= \frac{\tan^5 x}{5} + \frac{\tan^3 x}{3} + c$$

99. $\lim\limits_{x \to 0} \frac{\sin^2 ax}{bx} \cdot \frac{a}{a} \cdot \frac{x}{x}$

$$= \lim_{x \to 0}\left(\frac{\sin ax}{ax}\right)^2 \cdot \frac{a}{b} \cdot x = 0$$

100. Given that $f(x) = \tan x + e^{-2x} - 7x^3$

On differentiating w.r.t. x, we get

$f'(x) = \sec^2 x - 2e^{-2x} - 21x^2$

$\Rightarrow \quad f'(0) = \sec^2 0 - 2e^0 - 21 \times 0$

$= 1 - 2$

$= -1$

101. The equation of family of rectangular hyperbola is $xy = c^2$.

On differentiating w.r.t. x, we get

$$y + x\frac{dy}{dx} = 0$$

Thus, the order and degree of differential equation are 1 and 1 respectively.

102. Given that $f(x) = x^2 - 2x$

On differentiating w.r.t. x, we get

$f'(x) = 2x - 2$

$f(x)$ is increasing, if

$2x - 2 > 0$

$\Rightarrow \quad x > 1$

103. Put $\quad I = \int_0^1 x(1 - x)^n dx$

Again Put $1 - x = t$ and $dx = -dt$

$$\therefore \quad I = -\int_1^0 (1 - t)t^n\, dt$$

$$= \int_0^1 (t^n - t^{n+1})\, dt$$

$$= \left[\frac{t^{n+1}}{n+1} - \frac{t^{n+2}}{n+2}\right]_0^1$$

$$= \frac{1}{n+1} - \frac{1}{n+2}$$

$$= \frac{1}{(n+1)(n+2)}$$

104. If a and b be two distinct roots of a polynomial equation $f(x) = 0$. Then there exists at least one root lying between

a and b of the polynomial equation $f'(x) = 0$ [According to Rolle's Theorem]

105. Given that, $3^x + 3^y = 3^{x+y}$

On differentiating w.r.t. x, we get

$$3^x \log 3 + 3^y \log 3 \frac{dy}{dx} = 3^{(x+y)} \log 3 \left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \quad 3^x + 3^y \frac{dy}{dx} = 3^{x+y} + 3^{(x+y)} \frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx}(-3^{x+y} + 3^y) = 3^{x+y} - 3^x$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{3^x(3^y - 1)}{3^y(1 - 3^x)} = \frac{3^{x-y}(3^y - 1)}{(1 - 3^x)}$$

106. Suppose that, $I = \int \sec x^\circ \, dx$

$$= \int \sec \frac{\pi x}{180^\circ} dx$$

Put $\frac{\pi x}{180^\circ} = t$

$$\Rightarrow \quad dx = \frac{180^\circ}{\pi} dt$$

$$\therefore \quad I = \int \sec t \, dt \cdot \frac{180^\circ}{\pi}$$

$$= \frac{180^\circ}{\pi} \log \tan\left(\frac{\pi}{4} + \frac{\pi x}{360^\circ}\right) + c$$

107. Given that $P(x) = -3500 + (400 - x)x$

On differentiating w.r.t. x, we get

$$P'(x) = 400 - 2x$$

Put $P'(x) = 0$ for maxima or minima

$$400 - 2x = 0 \Rightarrow 2x = 400$$

$$\therefore \quad x = 200$$

Now, $P''(x) = -2x$

$\Rightarrow P''(200) = -400 < 0$

$P(x)$ is maximum at $x = 200$

Hence, required number of items = 200

108. Given equation $s = 64t - 16t^2$

$\therefore$ On differentiating w.r.t. t, we get

$$\frac{ds}{dt} = 64 - 32t$$

Put $\frac{ds}{dt} = 0$ for maximum height

$$64 - 32t = 0 \Rightarrow 32t = 64$$

$$\therefore \quad t = 2$$

$$\therefore \left(\frac{d^2s}{dt^2}\right)_{t=2} < 0$$

Hence, required time = 2 s

109. Given that $f(x) = 3x^2 + 6x - 9$

On differentiating w.r.t. x, we get

$$f'(x) = 6x + 6$$

$$\Rightarrow \quad f'(x) < 0, \forall (-\infty, -1)$$

$\therefore f(x)$ is decreasing in $(-\infty, -1)$.

110. Given that $f(x) = \sin^2 x^2$

$$\therefore \quad f'(x) = 2 \sin x^2 \cos x^2 \cdot 2x$$

$$= 4x \sin x^2 \cos x^2$$

111. Given that $f(x) = \cos x, g(x) = \log x$

and $y = gof(x)$

$$= g\{f(x)\}$$

$$= \log(\cos x)$$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{\cos x}(-\sin x) = -\tan x$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{\text{at } x=0} = -\tan 0 = 0$$

112. Given that $f(x) = \begin{cases} 3x - 4, 0 \le x \le 2 \\ 2x + \lambda, 2 < x \le 3 \end{cases}$

Also, $f(x)$ is continuous at $x = 2$

$$\therefore \quad \lim_{x \to 2} f(x) = f(2)$$

$$\Rightarrow \quad \lim_{x \to 2} (2x + \lambda) = 6 - 4$$

$$\Rightarrow \quad 4 + \lambda = 2$$

$$\therefore \quad \lambda = -2$$

113. Here, slope of line $x \cos\theta + y \sin\theta = 2$ is $-\cot\theta$ and slope of line $x - y = 3$ is 1.

Also, these lines are perpendicular to each other

$$\therefore \quad (-\cot\theta)(1) = -1$$

$$\Rightarrow \quad \cot\theta = 1 = \cot\frac{\pi}{4}$$

$$\therefore \quad \theta = \frac{\pi}{4}$$

114 If x-axis is a tangent to the given circle. Then circle touches the x-axis.

$$\sqrt{g^2 - k} = 0$$

$$\Rightarrow \quad g^2 = k$$

115. We know that, the sum of focal radii of any point on an ellipse is equal to length of major axis.

116. The required equation represents a straight line.

117. The equation of line perpendicular to given line

$$x + y - 11 = 0 \quad \ldots(i)$$

and $$-x + y + \lambda = 0 \quad \ldots(ii)$$

This equation passes through $(2, 3)$.

$$\Rightarrow \quad -2 + 3 + \lambda = 0$$

$$\therefore \quad \lambda = -1$$

$\therefore$ From Eq. (ii),

$$-x + y - 1 = 0$$

$$\therefore \quad y = x + 1$$

$\therefore$ From Eq. (i),

$$x + x + 1 - 11 = 0$$

$$\Rightarrow \quad 2x = 10$$

$$\therefore \quad x = 5$$

Hence, coordinates of foot of perpendicular from $(2, 3)$ to given line is $(5, 6)$.

118. We know that, the equation of x-axis is $y = 0$.

Thus, statement 1 is correct.

119. Put θ be the angle between given planes, then

$$\cos\theta = \frac{2 \times 1 + 1 \times (-1) + 1 \times 2}{\sqrt{4 + 1 + 1}\sqrt{1 + 1 + 4}}$$

$$= \frac{3}{6} = \frac{1}{2} = \cos\frac{\pi}{3}$$

$$\therefore \quad \theta = \frac{\pi}{3}$$

120. The equation of plane passing through x-axis is $x = a$.

This also passes through $(1, 2, 3)$

$$\therefore \quad x = 1$$