Solutions Permutations, Combinations and Probability

1. (a) The word RUMOUR consists of 6 letters in which each of 'R' and 'U' comes twice.

Here, $n = 6,\ p = 2,\ q = 2$

$\therefore$ Number of arrangements

$$= \frac{6!}{2!2!} \quad \text{(by short trick 3)}$$

$$= \frac{6\times5\times4\times3\times2\times1}{2\times2} = 180$$

2. (a) The word VISITING has 8 letters in which letters 'I' comes thrice.

$\therefore$ Number of arrangements $= \frac{8!}{3!}$

$$= 8\times7\times6\times\times4$$

$$= 6720$$

3. (a) The word 'SACRED' consists of 4 consonants (SCRD) and 2 vowels (AE). On keeping vowels together, we get SCRD (AE)

$\therefore$ Number of arrangements $= 5! \times 2!$

$$= 5\times4\times3\times2\times1\times2\times1$$

$$= 240$$

4. (d) Total number of letters is 7 and these letters can be written in 7! ways

$$= 7\times6\times5\times4\times3\times2\times1$$

$$= 5040 \text{ ways}$$

There are seven letters in the word THERAPY including 2 vowels (E,A) and five consonants (THRPY).

Now, consider two vowels as one letter.

We have, 6 letters which can be arranged in ${}^6P_6$ ways $= 6!$ ways

But vowels can be arranged in 2! ways.

Hence, the number of ways that all Vowels will come together $= 6! \times 2!$

$$= 1\times2\times3\times4\times5\times6\times2$$

$$= 1440$$

5. (a) Books on Economics are to be kept together.

Hence, we are to arrange 3 books on Management, 4 books on Statistics and one book on Economics.

These can be arranged in 8! ways. Again, 4 books on Economics can be arranged together in 4! ways.

$\therefore$ Total number of arrangements

$$= 8!\times4!$$

$$= 40320\times24 = 967680$$

6. (b) When teachers from each stream are to be included, then number of ways

$$= {}^4C_2\times{}^5C_2\times{}^3C_2$$

$$= \frac{4!}{(4-2)\times!}\times\frac{5!}{(5-2)!\times2!}\times\frac{3!}{(3-2)!\times2!}$$

$$= \frac{4\times3\times2!}{2!\times2\times1}\times\frac{5\times4\times3!}{3!\times2\times1}\times\frac{2!\times3}{1\times2!}$$

$$= 2\times3\times5\times2\times3 = 180$$

7. (d) When no teacher from the Commerce stream is to be included, then number of ways

$$= {}^9C_6 = \frac{9!}{(9-6)!} = \frac{9\times8\times7\times6!}{3\times2\times6!}$$

$$= 3\times4\times7 = 84$$

8. (c) When any teacher can be included in the committee, then number of ways

$$= {}^{12}C_6 \quad \text{(by short trick 1)}$$

$$= \frac{12!}{(12-6)!\times6!}$$

$$= \frac{12\times11\times10\times9\times8\times7\times6!}{6\times5\times4\times3\times2\times6!}$$

$$= 11\times2\times3\times2\times7$$

$$= 924$$

9. (e) The word CYCLE has 5 letters in which the letter 'C' comes twice.

$\therefore$ Number of arrangements $= \frac{5!}{2!}$

$$= \frac{5\times4\times3\times2\times1}{2\times1} = 60$$

10. (d) The word 'BANKING' consists of 7 letters in which 'N' comes twice.

$\therefore$ Number of arrangements $= \frac{7!}{2!}$

$$= \frac{7\times6\times5\times4\times3\times2}{2} = 2520$$

11. (d) Number of selections = Number of ways of selecting 2 men out of 5 men × Number of ways of selecting 1 woman out of 3 women

$$= {}^5C_2\times{}^3C_1 = \frac{5\times4}{1\times2}\times3 = 30$$

12. (b) The word PRIDE consists of 5 distinct letters.

$\therefore$ Number of arrangements $= 5!$

$$= 5\times4\times3\times2\times1 = 120$$

13. (b) 4 boys can be seated in a row in ${}^4P_4 = 4!$ ways.

Now, in the 5 gaps, 2 girls can be arranged ${}^2P_2$ ways.

Hence, the number of ways in which no two girls sit together

$$= 4!\times{}^5P_2 = 4\times3\times2\times5\times4 = 480$$

14. (c) There are 9 women and 8 men. A committee of 12, consisting of atleast 5 women, can be formed by choosing

(i) 5 women and 7 men

(ii) 6 women and 6 men

(iii) 7 women and 5 men

(iv) 8 women and 4 men

(v) 9 women and 3 men

$\therefore$ Total number of ways of forming the committee

$$= {}^6C_5\times{}^8C_7+{}^9C_6\times{}^8C_6+{}^9C_7$$

$\times ^8C_5 + ^9C_8 \times ^8C_4 + ^9C_9 \times ^8C_3$

$= 126 \times 8 + 84 \times 28 + 36 \times 56 + 9 \times 70 + 1 \times 56$

$= 1008 + 2352 + 2016 + 630 + 56 = 6062$

15. (d) Women are in majority in (iii), (iv) and (v) cases.

$\therefore$ Total number of such committees

$= ^9C_7 \times ^8C_5 + ^9C_8 \times ^8C_4 + ^9C_9 \times ^8C_3$

$= 36 \times 56 + 9 \times 70 + 1 \times 56$

$= 2016 + 630 + 56$

$= 2702$

16. (a) Total number of such committees

$= ^9C_5 \times ^8C_7$

$= 126 \times 8$

$= 1008$

17. (e) Total number of marbles in the urn = 15

$P(S)$ = Total possible outcomes

= Selection of 2 marbles at random out of 15 marbles $= ^{15}C_2 = \frac{15 \times 14}{1 \times 2} = 105$

$P(E)$ = Favourable outcomes

= Selection of 2 marbles out of 2 green marbles $= ^2C_2 = 1$

$\therefore$ Required probability $= \frac{(P)(E)}{P(S)} = \frac{1}{105}$

18. (c) $P(S) = ^{15}C_3 = \frac{15 \times 14 \times 13}{1 \times 2 \times 3} = 455$

$P(E)$ = Selection of 2 marbles out of 6 blue marbles and that of one marble out of 4 yellow marbles

$= ^6C_2 \times ^4C_1$

$= \frac{6 \times 5}{1 \times 2} \times 4 = 60$

$\therefore$ Required probability

$= \frac{P(E)}{P(S)} = \frac{60}{455} = \frac{12}{91}$

19. (b) $P(S) = ^{15}C_4$

$= \frac{15 \times 14 \times 13 \times 12}{1 \times 2 \times 3 \times 4} = 1365$

Let no yellow marble is selected.

$\therefore$ $P(E)$ = Selection of 4 marbles out of 11 marbles

$= ^{11}C_4 = \frac{11 \times 10 \times 9 \times 8}{1 \times 2 \times 3 \times 4} = 330$

$\therefore$ Required probability

$= 1 - \frac{330}{1365} = 1 - \frac{22}{91} = \frac{91 - 22}{91} = \frac{69}{91}$

20. (b) $P(S) = ^{15}C_2 = 105$

$P(E) = ^3C_2 + ^2C_2$

$= \frac{3 \times 2}{1 \times 2} + 1 = 3 + 1 = 4$

$\therefore$ Required probability $= \frac{4}{105}$

21. (c) $P(S) = ^{15}C_4 = 1365$

$P(E) = ^2C_1 \times ^6C_2 \times ^3C_1$

$= 2 \times 15 \times 3 = 90$

$\therefore$ Required probability

$= \frac{P(E)}{P(S)} = \frac{90}{1365} = \frac{6}{91}$

22. (d) Total possible outcomes = Number of ways of picking 3 marbles out of 12 marbles

$= n(S) = ^{12}C_3 = \frac{12 \times 11 \times 10}{1 \times 2 \times 3} = 220$

Favorable number of cases

$= n(E) = ^3C_3 + ^4C_3 = 1 + 4 = 5$

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{5}{220} = \frac{1}{44}$

23. (e) Total possible outcomes

$= n(S) = ^{12}C_2 = \frac{12 \times 11}{1 \times 2} = 66$

Favorable number of cases $= n(E)$

$= ^4C_2 = \frac{4 \times 3}{1 \times 2} = 6$

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{6}{66} = \frac{1}{11}$

24. (b) Total possible outcomes $= n(S)$

$= ^{12}C_3 = 220$

Favorable number of cases $= n(E)$

= Number of ways of picking 3 marbles (none is blue) out of 7 marbles

$= ^7C_3 = \frac{7 \times 6 \times 5}{1 \times 2 \times 3} = 35$

$\therefore$ Required probability

$= 1 - \frac{35}{220} = 1 - \frac{7}{44} = \frac{37}{44}$

25. (b) Number of possible outcomes,

$n(S) = ^7C_3 = \frac{7 \times 6 \times 5}{1 \times 2 \times 3} = 35$

Favorable number of cases $= ^3C_3 + ^4C_3$

$= 1 + 4 = 5$

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{5}{35} = \frac{1}{7}$

26. (b) Total possible outcomes $= n(S)$

= Selection fo 4 marbles our of 15 marbles

$= ^{15}C_4 = \frac{15 \times 14 \times 13 \times 12}{1 \times 2 \times 3 \times 4} = 1365$

When no marble is blue, favourable number of cases = $n(E)$ = Selection of 4 marbles out of 11 marbles

$= ^{11}C_4 = \frac{11 \times 10 \times 9 \times 8}{1 \times 2 \times 3 \times 4} = 1365$

$\therefore$ Required Probability $= 1 - \frac{n(E)}{n(S)}$

$= 1 - \frac{330}{1365} = 1 - \frac{22}{91} = \frac{69}{91}$

27. (e) Total possible outcomes $= n(S)$

$= ^{15}C_2 = \frac{15 \times 14}{1 \times 2} = 105$

Favourable number of cases $= n(S)$

= Selection of 2 marbles out of 6 marbles

$$= {}^6C_2 = \frac{6 \times 5}{1 \times 2} = 15$$

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{15}{105} = \frac{1}{7}$

28. (c) Total possible outcomes $= n(S)$

$$= {}^{15}C_3 = \frac{15 \times 14 \times 13}{1 \times 2 \times 3} = 455$$

Favourable number of cases $= n(S)$

$$= {}^4C_2 \times {}^3C_1 = \frac{4 \times 3}{1 \times 2} \times 3 = 6 \times 3 = 18$$

$\therefore$ Required probability $= \frac{18}{455}$

29. (a) Total possible outcomes $= n(S)$

$$= {}^{15}C_4$$
$$= 1365$$

Favourable number of cases $= n(E)$

$$= {}^2C_1 \times {}^4C_2 \times {}^6C_1$$
$$= 2 \times \frac{4 \times 3}{1 \times 2} \times 6 = 72$$

$\therefore$ Required probability $= \frac{72}{1365} = \frac{24}{455}$

30. (d) total possible outcome $= n(S)$

$$= {}^{15}C_2$$
$$= 105$$

Favourable number of cases $= n(E)$

$$= {}^2C_2 + {}^3C_2$$
$$= 1 + 3 = 4$$

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{4}{105}$

31. (b) Total number of balls in the basket $= 15$

Exhaustive number of cases $= {}^{15}C_2$

$$= \frac{15 \times 14}{1 \times 2} = 105$$

Favourable number of outcomes

$$= {}^4C_2 + {}^3C_2$$
$$= 6 + 3 = 9$$

$\therefore$ Required probability $= \frac{9}{105} = \frac{3}{35}$

32. (a) Exhaustive number of cases = Number of ways of selection 5 balls out of 15 balls of 9 balls without blue balls = ${}^9C_5$

$\therefore$ Required probability $= 1 - \frac{{}^9C_5}{{}^{15}C_5}$

$$= 1 - \frac{9 \times 8 \times 7 \times 6 \times 5}{15 \times 14 \times 13 \times 12 \times 11}$$
$$= 1 - \frac{6}{143} = \frac{137}{143}$$